

## SM2H 9.1N – Degrees and Radians

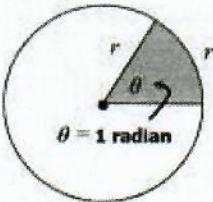
Fraction Review:

1.  $\frac{2\pi}{5} \cdot \frac{1}{4}$

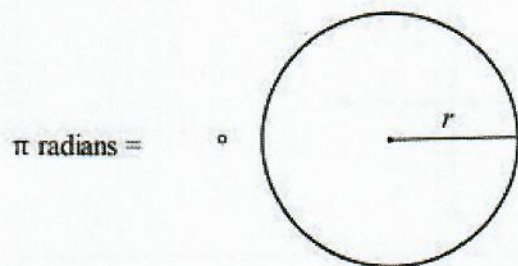
2.  $\frac{3\pi}{8} \cdot \frac{4}{\pi}$

3.  $\frac{5x}{3} - \frac{x}{4}$

4.  $\frac{\pi}{6} + \frac{\pi}{3}$

<p><b>Radians vs. Degrees</b></p> 	<p>A <b>radian</b> is a unit of angle measure based on arc length. One radian is defined as the measure of the angle formed when the radius is equivalent to the length of the intercepted arc. Recall that the circumference of a circle is <math>2\pi r</math>, therefore:</p> <p><math>360^\circ = \underline{\hspace{2cm}}</math>; <math>180^\circ = \underline{\hspace{2cm}}</math></p>	
	<p><b>Converting Degrees → Radians</b></p>	<p><b>Converting Radians → Degrees</b></p>
	<p>Radians = Degrees <math>\cdot \left( \frac{\pi \text{ radians}}{180} \right)</math></p>	<p>Degrees = Radians <math>\cdot \left( \frac{180}{\pi \text{ radians}} \right)</math></p>

What is a radian? [https://commons.wikimedia.org/wiki/File:Circle\\_radians.gif#/media/File:Circle\\_radians.gif](https://commons.wikimedia.org/wiki/File:Circle_radians.gif#/media/File:Circle_radians.gif)



Convert each degrees to radians and radians to degrees:

1.  $45^\circ$

2.  $\frac{2\pi}{3}$

3.  $-\frac{11\pi}{6}$





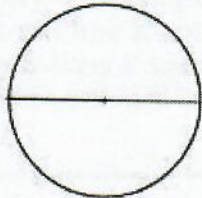
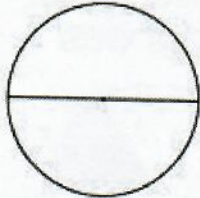
4.  $-935^\circ$

5.  $\frac{7\pi}{4}$

6.  $80^\circ$

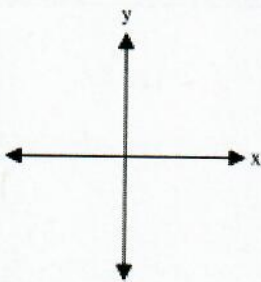
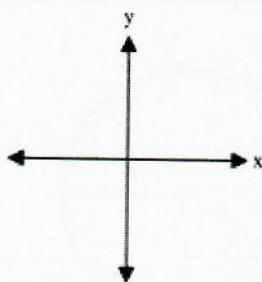
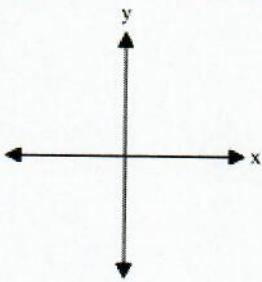
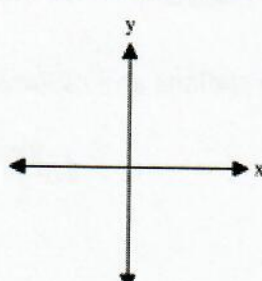
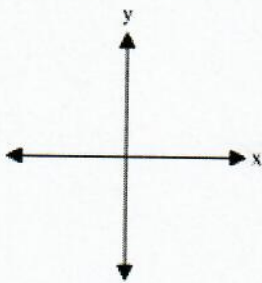
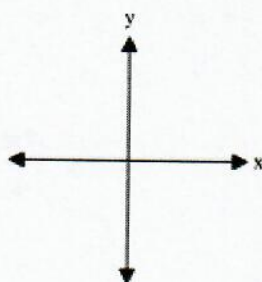
# Coterminal and Reference Angles

A. Shade the appropriate portion of the semi-circle.

1. $\frac{1}{3}$ 	2. $\frac{2}{5}$ 	3. $\frac{\pi}{4}$ $\frac{4\pi}{4}$ 
4. $\frac{5\pi}{6}$ $\frac{6\pi}{6}$ 	5. $\frac{9\pi}{8}$ $\frac{8\pi}{8}$ 	6. $\frac{3\pi}{2}$ $\frac{2\pi}{2}$ 

## B. Drawing Angles

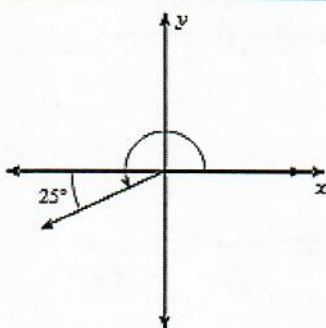
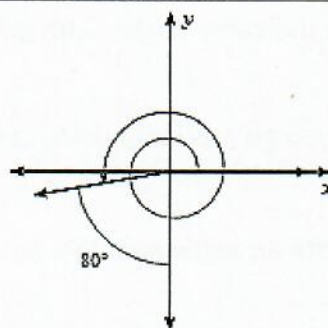
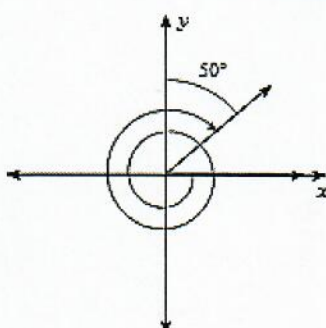
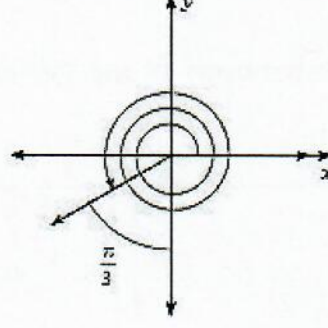
- Draw and Label the Given angle.
- Draw and Label the *reference angle*. (Remember- A **reference angle** is formed by the *terminal side* of the angle and the *x-axis*. This means the angle will always be less than \_\_\_\_\_.)

1. $60^\circ$ 	2. $-380^\circ$ 
3. $\frac{3\pi}{4}$ 	4. $-\frac{11\pi}{6}$ 
5. $-525^\circ$ 	6. $\frac{67\pi}{18}$ 

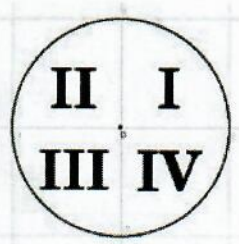


### C. Measures of Angles

Find the measure of each angle and then find the reference angle.

<p>1.</p> 	<p>2.</p> 
<p>3.</p> 	<p>4.</p> 

### D. Determine the quadrant of each angle.

	<p>1. <math>480^\circ</math></p>	<p>2. <math>\frac{9\pi}{4}</math></p>
	<p>3. <math>-\frac{7\pi}{3}</math></p>	<p>4. <math>-256^\circ</math></p>
<p>5. <math>\frac{13\pi}{6}</math></p>	<p>6. <math>-\frac{19\pi}{4}</math></p>	<p>7. <math>\frac{24\pi}{6}</math></p>

**E. Finding *coterminal angles* between  $0^\circ$  and  $360^\circ$  or 0 and  $2\pi$ .**

Watch youtube video : <https://www.youtube.com/watch?v=A8NoBcYQJ1U>

Coterminal angles are the same angles with different measures. Coterminal angles share the \_\_\_\_\_ side of the angle.

What happens when you go around a circle more than once?

Fill in the blank to create an angle equal to  $2\pi$ .  $2\pi = \frac{\quad}{8} = \frac{\quad}{3} = \frac{\quad}{5} = \frac{\quad}{16}$

In fractions that are equal to  $2\pi$ , **the numerator must be double the denominator.**

You can tell if the angle is bigger than  $2\pi$  if the numerator is **more** than double the denominator.

Find a coterminal angle between  $0^\circ$  and  $360^\circ$  or 0 and  $2\pi$ .

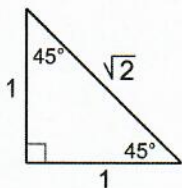
1. $580^\circ$	2. $-92^\circ$	3. $\frac{9\pi}{4}$
4. $-\frac{2\pi}{3}$	5. $-225^\circ$	6. $\frac{116\pi}{45}$



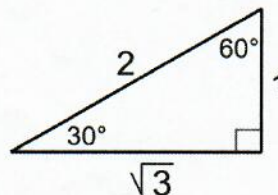
## SM2H 9.2 Notes – The Unit Circle

### Review Special Right Triangles

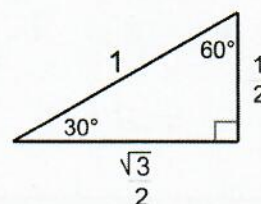
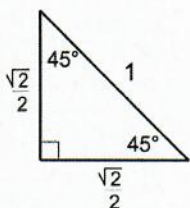
In a  $45^\circ - 45^\circ - 90^\circ$  triangle,  
the sides are in the ratio  $1:1:\sqrt{2}$ .



In a  $30^\circ - 60^\circ - 90^\circ$  triangle,  
the sides are in the ratio  $1:\sqrt{3}:2$ .



Now let's make the hypotenuse 1 so we'll be able to use the special right triangles in the Unit Circle.  
The Unit Circle is a circle where the radius is equal to 1 unit. Using the rules of special right triangles, the new side lengths are:

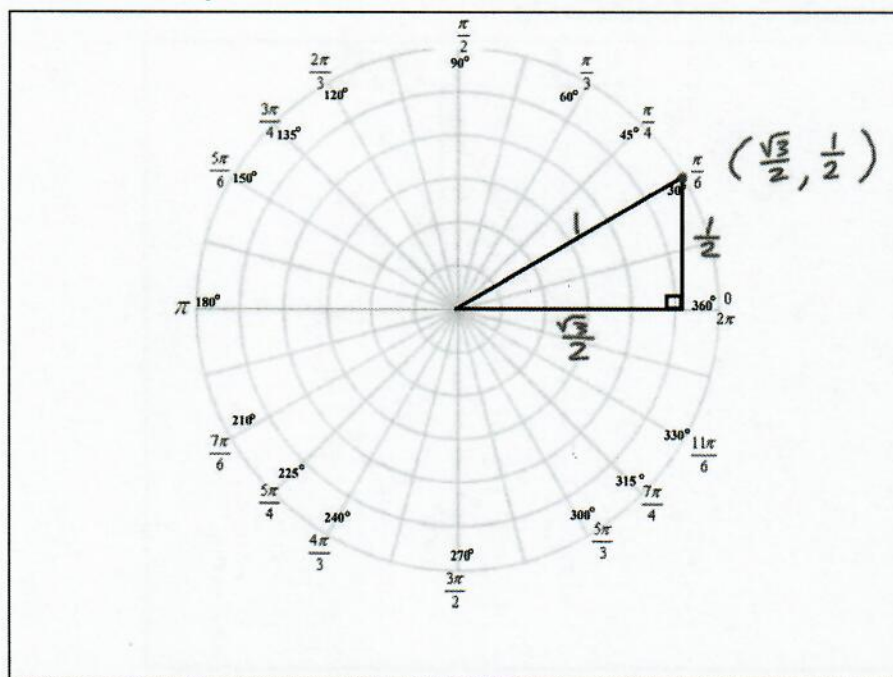


**\*\*Please watch this video on understanding the Unit Circle\*\***

<https://www.youtube.com/watch?v=n6L7VkdMv2g>

### The $30^\circ$ triangle in the Unit Circle:

The above ratios of the  $30^\circ - 60^\circ - 90^\circ$  triangle were used to fill in the side lengths and coordinates of the  $30^\circ$  triangles in the Unit Circle.

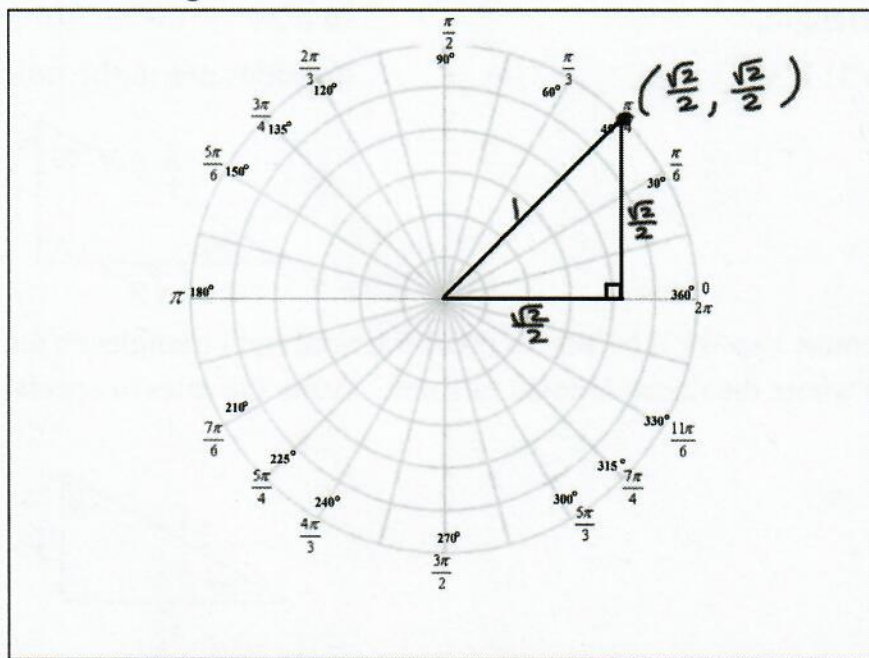


Sketch in the other three  $30^\circ$  triangles (reference angles) including their side lengths and coordinates by reflecting the above triangle over the x and y axes. Draw a box connecting the coordinates to show that all of the coordinates are related.



## The 45° triangle in the Unit Circle:

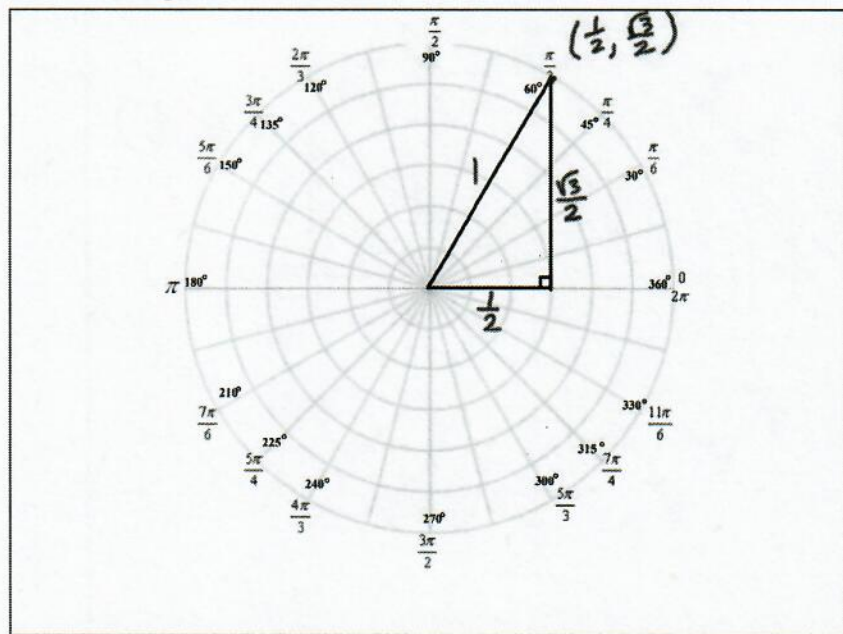
The above ratios of the 45° – 45° – 90° triangle were used to fill in the side lengths and coordinates of the 45° triangle in the Unit Circle.



Sketch in the other three 45° triangles (reference angles) including their side lengths and coordinates by reflecting the above triangle over the x and y axes. Draw a box connecting the coordinates to show that all of the coordinates are related.

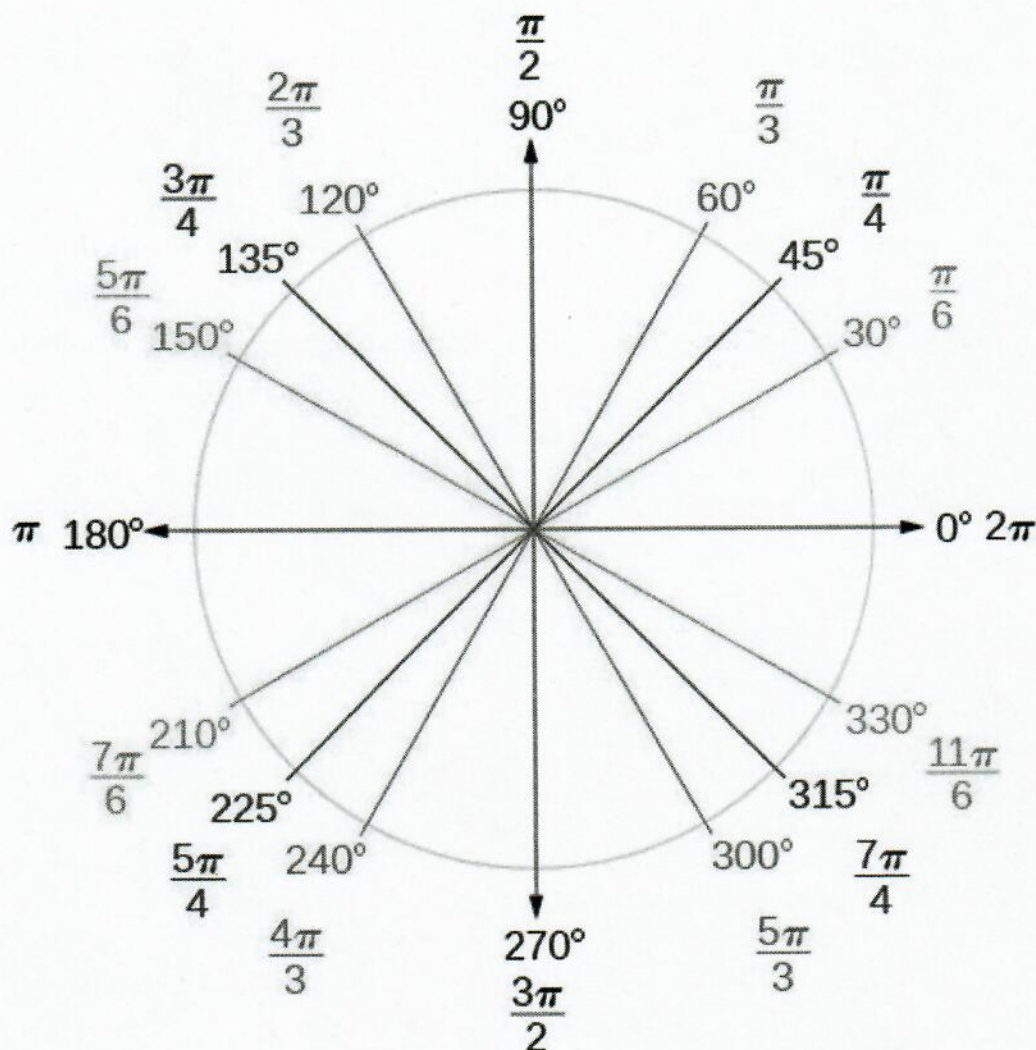
## The 60° triangle in the Unit Circle:

The above ratios of the 30° – 60° – 90° triangle were used to fill in the side lengths and coordinates of the 60° triangle in the Unit Circle.



Sketch in the other three 60° triangles (reference angles) including their side lengths and coordinates by reflecting the above triangle over the x and y axes. Draw a box connecting the coordinates to show that all of the coordinates are related.

Fill in the coordinates of the first quadrant of the Unit Circle and then reflect those coordinates over the x and y axes to find the coordinates of all the other angles on the unit circle.



Remember from the video that **in the Unit Circle**  $(x, y) = (\cos\theta, \sin\theta)$

$$x = \cos\theta \quad y = \sin\theta \quad \frac{y}{x} = \tan\theta$$

Using the Unit Circle find the exact value of:

a)  $\cos 60^\circ =$

d)  $\sin 270^\circ =$

g)  $\tan \frac{2\pi}{3} =$

b)  $\sin \frac{4\pi}{3} =$

e)  $\cos \frac{5\pi}{4} =$

h)  $\tan 90^\circ =$

c)  $\tan 45^\circ =$

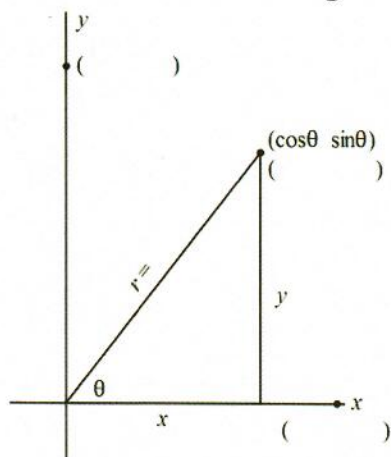
f)  $\tan \frac{\pi}{6} =$

i)  $\tan \frac{7\pi}{6} =$





## SM2H 9.3 Notes Using the Unit Circle

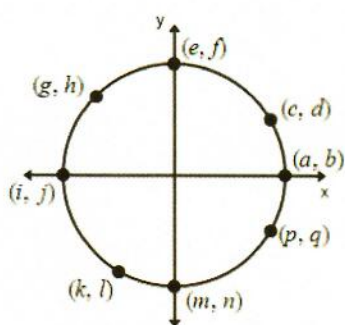


$$\cos \theta = \quad \sec \theta =$$

$$\sin \theta = \quad \csc \theta =$$

$$\tan \theta = \quad \cot \theta =$$

Refer to the diagram below. Give the letter or letters that could stand for the function value.



1.  $\cos 180^\circ$

2.  $\tan 270^\circ$

3.  $\sin \frac{11\pi}{6}$

4.  $\sec 270^\circ$

5.  $\csc 30^\circ$

6.  $\cos 135^\circ$

7.  $\cot 330^\circ$

8.  $\sec \frac{\pi}{2}$

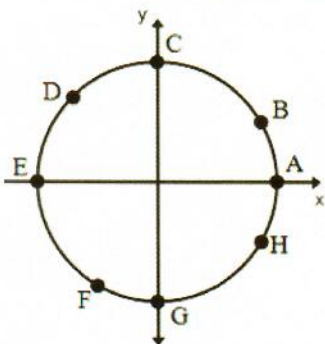
9.  $\tan \frac{4\pi}{3}$

11.  $\cos -\frac{11\pi}{6}$

12.  $\sin -\frac{2\pi}{3}$

13.  $\cot -\frac{5\pi}{4}$

Refer to the diagram below. For the indicated point, tell if the value for  $\sin \theta$  or  $\cos \theta$  is positive, negative, neither (zero), or undefined.



10.  $\sin D$

11.  $\sec B$

12.  $\cos G$

13.  $\cot A$

14.  $\cos F$

18.  $\cot F$

16.  $\sin H$

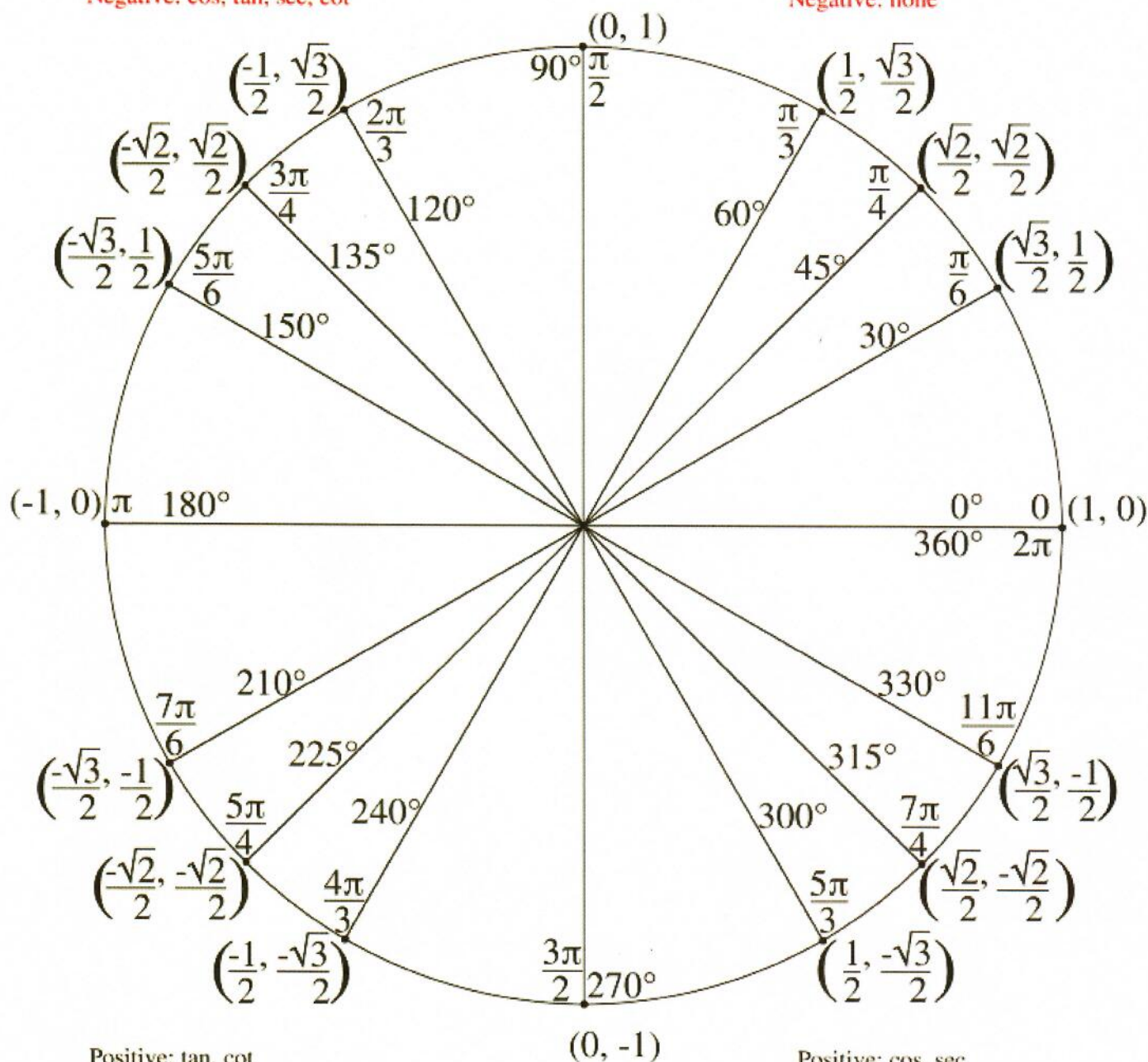
17.  $\csc H$

15.  $\cos A$

# The Unit Circle

Positive: sin, csc  
Negative: cos, tan, sec, cot

Positive: sin, cos, tan, sec, csc, cot  
Negative: none



Positive: tan, cot  
Negative: sin, cos, sec, csc

Positive: cos, sec  
Negative: sin, tan, csc, cot



Use the unit circle to find the exact value of each trigonometric function.

1.  $\sin 30^\circ$

2.  $\cos 30^\circ$

3.  $\tan 30^\circ$

4.  $\csc 30^\circ$

5.  $\sec 30^\circ$

6.  $\cot 30^\circ$

7.  $\cos \frac{5\pi}{6}$

8.  $\csc \frac{3\pi}{2}$

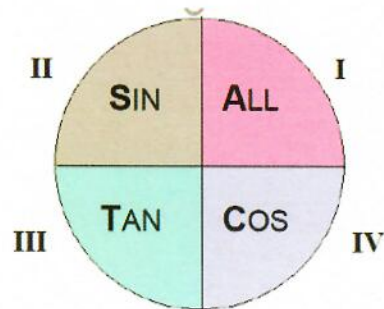
9.  $\cot \frac{11\pi}{6}$

10.  $\tan 90^\circ$

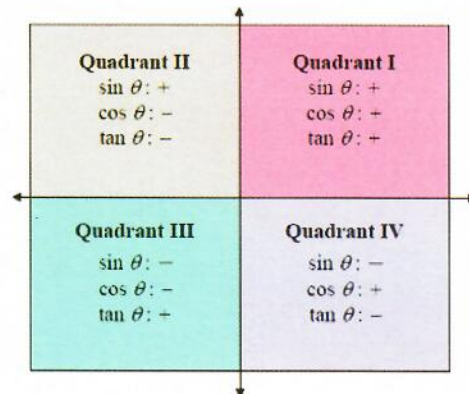
11.  $\sec \frac{5\pi}{3}$

12.  $\cot \frac{\pi}{2}$

**Signs of the Trig Functions: Where are the trig functions Positive?**



**All Students Take Calculus**



**Name the Quadrant in which the angle lies.**

1)  $\tan \theta > 0, \sin \theta < 0$

A) I

B) II

C) III

D) IV

2)  $\cos \theta < 0, \sin \theta < 0$

A) I

B) II

C) III

D) IV

3)  $\sin \theta > 0, \cos \theta < 0$

A) I

B) II

C) III

D) IV

4)  $\tan \theta < 0, \cos \theta > 0$

A) I

B) II

C) III

D) IV

5)  $\sin \theta > 0, \cos \theta > 0$

A) I

B) II

C) III

D) IV

6)  $\cos \theta < 0, \tan \theta < 0$

A) I

B) II

C) III

D) IV

7)  $\tan \theta < 0, \sin \theta < 0$

A) I

B) II

C) III

D) IV

8)  $\cos \theta > 0, \sin \theta < 0$

A) I

B) II

C) III

D) IV

**Find the exact measures of the angles  $[0^\circ, 360^\circ)$  using the unit circle.**

1.  $\sin \theta = \frac{\sqrt{3}}{2}$

2.  $\tan \theta = 0^\circ$

3.  $\cos \theta = -\frac{\sqrt{2}}{2}$

4.  $\sin \theta = -1$



## SM2H Notes 9.4 Trigonometric Identities

### Trigonometry Function Identities

#### Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

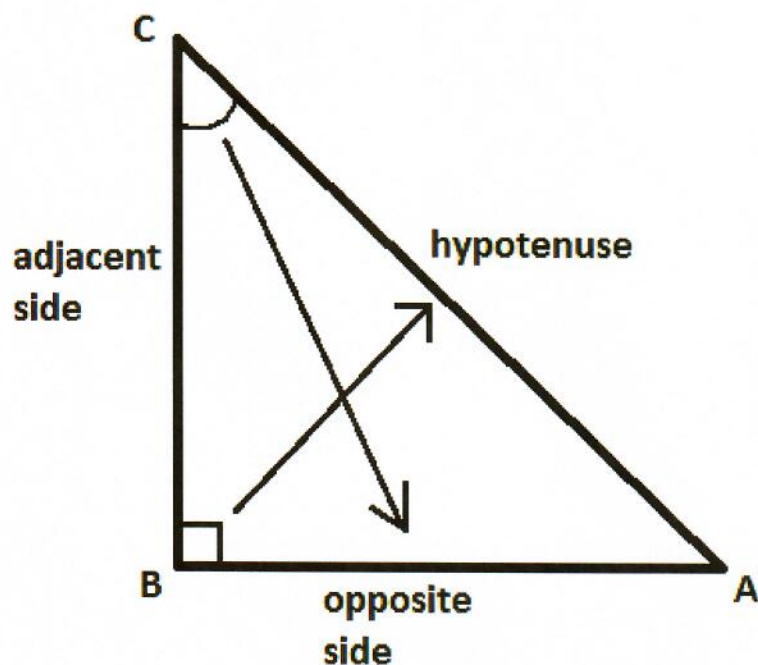
#### Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

### SOH CAH TOA



$$\sin c = \frac{\text{opposite}}{\text{hypotenuse}} =$$

$$\cos c = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan c = \frac{\text{opposite}}{\text{adjacent}} =$$

$$\cot c = \frac{\text{adjacent}}{\text{opposite}}$$

$$\sec c = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\csc c = \frac{\text{hypotenuse}}{\text{opposite}}$$

#### Review

1. Multiply  $\frac{15}{16} \cdot \frac{4}{25}$

2. Add  $\frac{3}{8} + \frac{5}{6}$

3. Divide  $\frac{\frac{8}{25}}{\frac{2}{5}}$

**Prove the identities, using opposite, adjacent, and hypotenuse.**

4.  $\sin \theta = \frac{1}{\csc \theta}$

5.  $\tan \theta = \frac{\sin \theta}{\cos \theta}$

**Write each expression in terms of sine and/or cosine, then simplify.**

6.  $\sec x \cdot \sin x$

7.  $\frac{\tan x}{\sin x}$

**Prove the trigonometric identities.**

8.  $\cos x \cdot \sec x = 1$

9.  $\cos \theta \cdot \csc \theta = \cot \theta$



## SM2H Notes 9.5 Trigonometric Identities Day 2

### Review Concepts

1.  $\frac{1}{\frac{3}{5}}$

2.  $(x + 6)(x - 6)$

3.  $\frac{3}{x} + \frac{5}{y}$

4.  $\sin x + \sin x =$

5.  $\sin x \cdot \sin x =$

6.  $(1 - \cos x)(1 + \cos x) =$

7.  $\tan^2 x = \tan x \cdot \tan x$  so  $\tan x \cdot \tan x = \frac{\sin x \cdot \sin x}{\cos x \cdot \cos x}$  or  $\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$

### Factor out the GCF

8.  $5x^3 - 25x^2 + 5x$

9.  $\sin^2 x + \sin x$

The Pythagorean Identity:

$$\sin^2 x + \cos^2 x = 1$$

If we divide each term of the fundamental identity by  $\sin^2 x$  or  $\cos^2 x$ , we can derive two more identities. These are called Pythagorean Identities because they are related to the Pythagorean Theorem:

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan^2 x + 1 = \sec^2 x$$

**You can rewrite the Pythagorean Identities by using the addition property of addition and subtraction.**

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

### **A General Strategy for Proving Identities**

1. Work on the more complicated side first.
2. Rewrite the side you are working with in terms of sines and cosines only.
3. Multiply the numerator and denominator of one rational expression by either the numerator or denominator of the other.
4. Write a single rational expression as a sum of two rational expressions.
5. Combine a sum of two rational expressions into a single rational expression.
6. If both sides simplify to a third expression, then the equation is an identity.
7. NO solving (NO whatever you do to one side do to the other)

**Prove each identity.**

1.  $\frac{\tan x}{\sin x} = \sec x$

2.  $\sin x + \sin x \cdot \cot^2 x = \csc x$

3.  $\sin \theta \cdot \cos \theta \cdot \tan \theta = 1 - \cos^2 \theta$

4.  $\cos x + \sin x \cdot \tan x = \sec x$

5.  $\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \csc x$

6.  $\frac{\sin^2 x}{\cos x} = \sec x - \cos x$