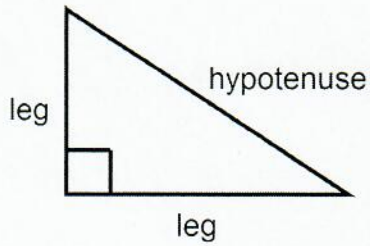
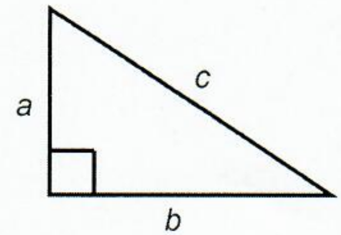
8.1 The Pythagorean Theorem/Trigonometric Ratios

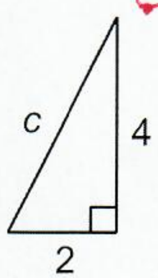
a right triangle, $a^2 + b^2 = c^2$, or $\mathbf{leg}^2 + \mathbf{leg}^2 = \mathbf{hypotenuse}^2$.





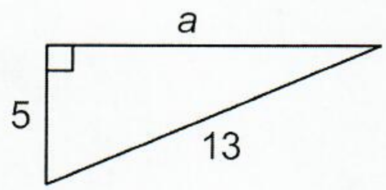
★ **The hypotenuse (the longest side – the one across from the right angle) should always be by itself on one side of the equation.**

To find the length of the hypotenuse:



$$2^2 + 4^2 = c^2$$
$$4 + 16 = c^2$$
$$c^2 = 20$$
$$c = \sqrt{20}$$
$$c = 2\sqrt{5} \approx 4.47$$

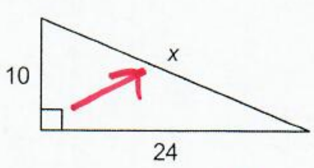
To find the length of a leg:



$$a^2 + 5^2 = 13^2$$
$$a^2 + 25 = 169$$
$$a^2 = 169 - 25$$
$$a^2 = 144$$
$$a = \sqrt{144}$$
$$a = 12$$

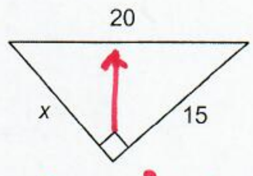
Examples: Find the length of the missing side of each triangle.

a)



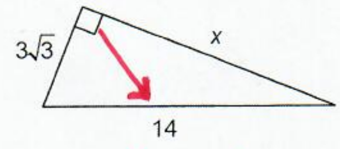
$10^2 + 24^2 = x^2$
$100 + 576 = x^2$
$\sqrt{676} = \sqrt{x^2}$
$\boxed{26 = x}$

b)



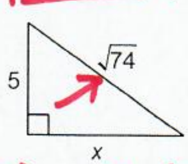
$x^2 + 15^2 = 20^2$
$x^2 + 225 = 400$
$x^2 = 175$
$\boxed{x = 13.23}$

c)



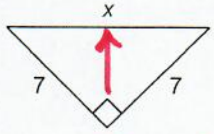
$(3\sqrt{3})^2 + x^2 = 14^2$
$27 + x^2 = 196$
$x^2 = 169$
$\boxed{x = 13}$

d)



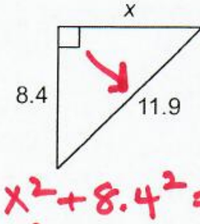
$5^2 + x^2 = (\sqrt{74})^2$
$25 + x^2 = 74$
$x^2 = 49$
$\boxed{x = 7}$

e)



$7^2 + 7^2 = x^2$
$49 + 49 = x^2$
$98 = x^2$
$\boxed{9.90 = x}$

f)



$x^2 + 8.4^2 = 11.9^2$
$x^2 + 70.56 = 141.61$
$x^2 = 71.05$
$\boxed{x = 8.43}$

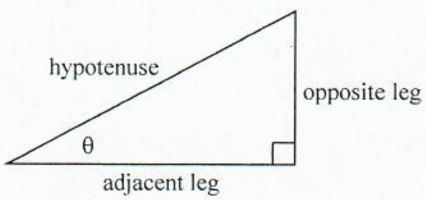
Right Triangle Trigonometric Ratios

Trigonometry: The study of the relationships among the sides and angles of right triangles.

Trigonometric Ratio: A ratio of the lengths of two sides of a right triangle. The three main trigonometric rations are sine (sin), cosine (cos), and tangent (tan).

If θ is an acute angle of a right triangle, "adj" is the length of the leg adjacent (next to) θ,

"opp" is the length of the leg opposite θ, and "hyp" is the length of the hypotenuse, then:



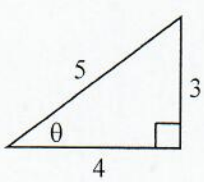
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

<u>A common way to remember this is SOH-CAH-TOA</u>

Three other trig ratios: $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$

Examples: Find the exact values of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$ and $\cot\theta$.

a)



opp= 3

adj= 4

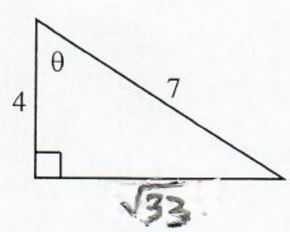
hyp= 5

$sin\theta = \frac{3}{4}$ $csc\theta = \frac{4}{3}$

$cos\theta = \frac{4}{5}$ $sec\theta = \frac{5}{4}$

$tan\theta = \frac{3}{4}$ $cot\theta = \frac{4}{3}$

b)



opp= $\sqrt{33}$

adj= 4

hyp= 7

$sin\theta = \sqrt{33}$ $csc\theta = \frac{7}{\sqrt{33}} \cdot \frac{\sqrt{33}}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$

$cos\theta = \frac{4}{7}$ $sec\theta = \frac{7}{4}$

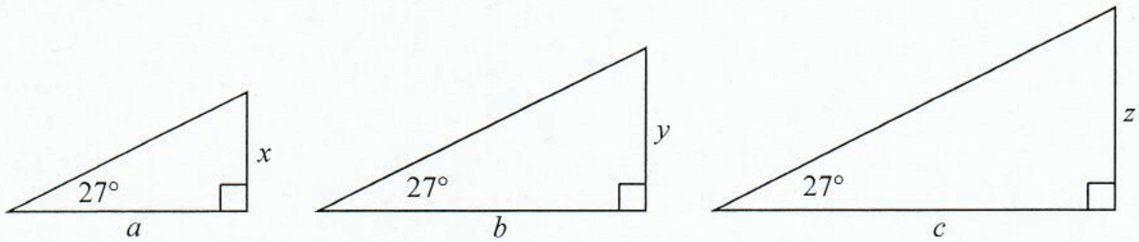
$tan\theta = \frac{\sqrt{33}}{4}$ $cot\theta = \frac{4}{\sqrt{33}} \cdot \frac{\sqrt{33}}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$

$4^2 + x^2 = 7^2$

$16 + x^2 = 49$

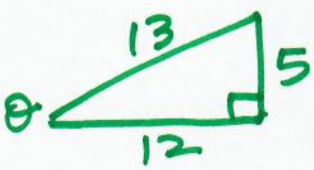
$x^2 = 33$ $\sqrt{33}$ $x = 5.74$

No matter how big the triangle is, the values of the trigonometric functions for a certain size angle will remain the same. For example, in the diagram below, $\tan 27° = \frac{x}{a} = \frac{y}{b} = \frac{z}{c}$. The value of the tangent is the same in all three triangles even though they are different sizes. The same is true for the sine and cosine.



Examples: Draw and label a triangle, find the length of the missing side, and find the requested values.

Find *sinθ*, *cosθ*, *tanθ*, *cscθ*, *secθ*, and *cotθ* if $\sin\theta = \frac{5}{13}$



$5^2 + x^2 = 13^2$

$25 + x^2 = 169$

$x^2 = 144$

$x = 12$

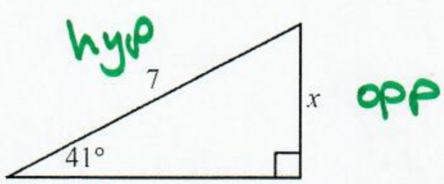
$\sin\theta = \frac{5}{13}$ $\csc\theta = \frac{13}{5}$

$\cos\theta = \frac{12}{13}$ $\sec\theta = \frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\cot\theta = \frac{12}{5}$

Identify which trigonometric ratio is needed to solve for the missing side. Write the correct equation, then solve. Round to the nearest hundredths.

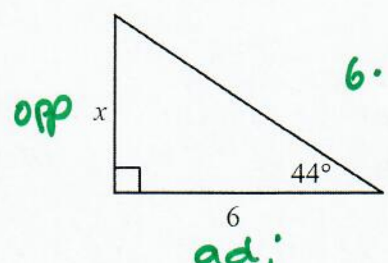
a)



$7 \cdot \sin 41° = \frac{x}{\cancel{7}} \cdot \cancel{7}$

$\boxed{4.59 = x}$

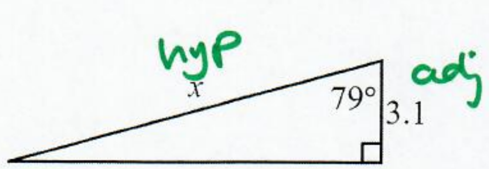
b)



$6 \cdot \tan 44° = \frac{x}{\cancel{6}} \cdot \cancel{6}$

$\boxed{5.79 = x}$

c)

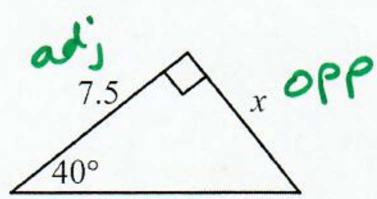


$x \cdot \cos 79° = \frac{3.1}{\cancel{x}} \cdot \cancel{x}$

$\frac{x \cancel{\cos 79°}}{\cancel{\cos 79°}} = \frac{3.1}{\cos 79°}$

$\boxed{x = 16.25}$

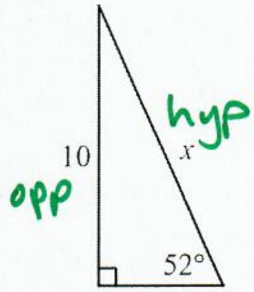
d)



$7.5 \cdot \tan 40° = \frac{x}{\cancel{7.5}} \cdot \cancel{7.5}$

$\boxed{6.29 = x}$

e)

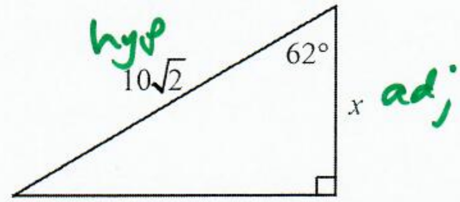


$x \cdot \sin 52° = \frac{10}{x} \cdot \cancel{x}$

$\frac{x \sin 52}{\sin 52} = \frac{10}{\sin 52}$

$\boxed{x = 12.69}$

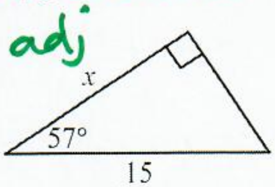
f)



$10\sqrt{2} \cdot \cos 62 = \frac{x}{10\sqrt{2}} \cdot 10\sqrt{2}$

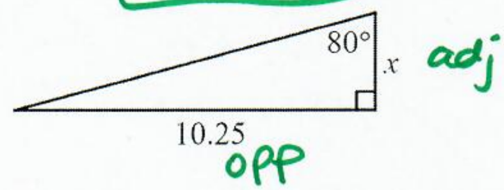
$\boxed{6.64 = x}$

g)



$15 \cdot \cos 57° = \frac{x}{15} \cdot 15$

h)



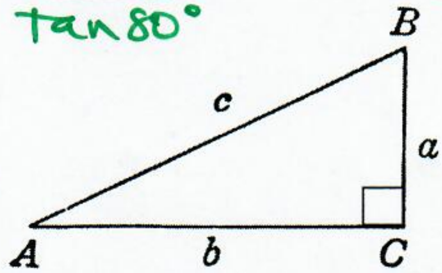
$x \cdot \tan 80° = \frac{10.25}{x} \cdot \cancel{x}$

$\frac{x \tan 80°}{\tan 80°} = \frac{10.25}{\tan 80°}$

$\boxed{x = 1.81}$

Triangle Notation:

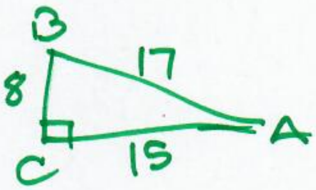
When labeling a triangle, there are a few things to remember.



1. **Capital letters refer to angles**
2. **Lower case letters are the side opposite their capital letter angle.**
3. **In a right triangle, "C" is 90°and "c" is the hypotenuese**

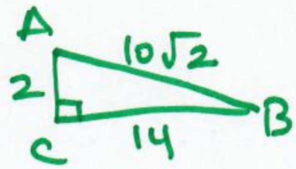
In each triangle ABC, angle C is a right angle. Find the value of the trig function indicated.

a) Find Find cosA if $a = 8$, $c = 17$, $b = 15$.



$\cos A = \frac{15}{17}$

b) Find cscA if $a = 14$, $b = 2$, and $c = 10\sqrt{2}$.



$\sin A = \frac{14}{10\sqrt{2}}$

$\boxed{\csc A = \frac{10\sqrt{2}}{14}}$

Examples: Use a calculator to approximate each value to four decimal places. Make sure your calculator is in degree mode.

a) sin 120°

0.8660

b) cos 350°

0.9848

c) tan −30°

−0.5774

d) cot 280°

−0.1763 $\frac{1}{\tan 280°}$

e) sec 360°

1 $\frac{1}{\cos 360°}$

e) csc 360°

Undefined $\frac{1}{\sin 360°}$

8.2 Inverse Trigonometric Functions

★ **REMEMBER from 9.1 TrigFunction(angle/theta)=Ratio**

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \quad \cos\theta = \frac{\text{adj}}{\text{hyp}} \quad \tan\theta = \frac{\text{opp}}{\text{adj}} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}} \quad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Inverse Functions:

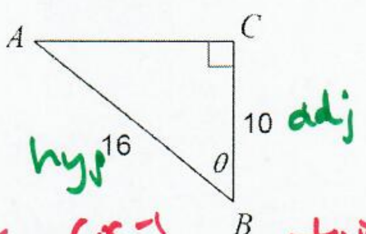
- The inverse sine of x ($\sin^{-1} x$) is the angle whose sine is x. If $\sin\theta = x$, then $\theta = \sin^{-1} x$.
- The inverse cosine of x ($\cos^{-1} x$) is the angle whose cosine is x. If $\cos\theta = x$, then $\theta = \cos^{-1} x$.
- The inverse tangent of x ($\tan^{-1} x$) is the angle whose tangent is x. If $\tan\theta = x$, then $\theta = \tan^{-1} x$.

★ **Use inverse functions when you know the sine, cosine, or tangent of an angle and <u>want to know how big the angle is.</u>**

Use a calculator to find each angle measure to the nearest degree.

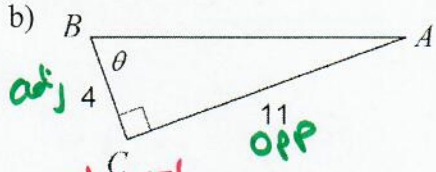
1. $\sin^{-1}$ ~~sin~~ C = .2250 — $13°$
2. $\cos^{-1}$ ~~cos~~ B= .1045 — $84°$
3. $\tan^{-1}$ ~~tan~~ A = 1.2799 — $52°$

Examples: Find the measure of the indicated angle to the nearest tenth of a degree.

a) 

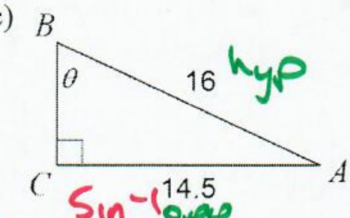
$\cos^{-1}$ ~~cos~~ $\theta = \frac{10}{16}$

$\theta = 51.3°$

b) 

$\tan^{-1}$ ~~tan~~ $\theta = \frac{11}{4}$

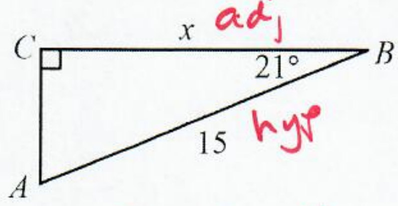
$\theta = 70.0°$

c) 

$\sin^{-1}$ ~~sin~~ $\theta = \frac{14.5}{16}$

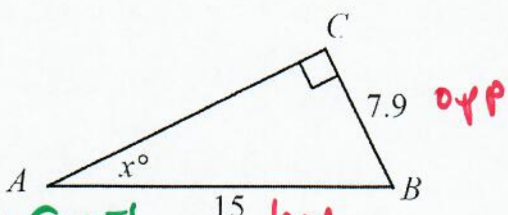
$\theta = 65°$

Find the value of x in each figure. Round your answer to the nearest hundredth.

a) 

$15 \cdot \cos 21° = \frac{x}{\cancel{15}} \cdot \cancel{15}$

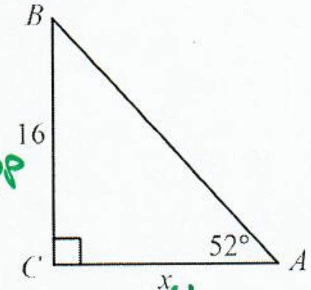
$\boxed{14 = x}$

b) 

$\sin^{-1}$ ~~sin~~ $x = \frac{7.9}{15}$

$\boxed{x = 31.78°}$

c)



opp

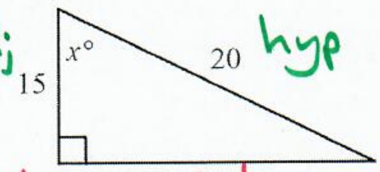
adj

$x \cdot \tan 52° = \frac{16}{\cancel{x}} \cdot \cancel{x}$

$\frac{x \tan 52°}{\tan 52°} = \frac{16}{\tan 52°}$

$\boxed{x = 12.5}$

d)



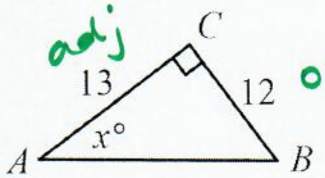
adj

hyp

$\cancel{\cos^{-1}}\ \cancel{\cos}\ x = \cos^{-1} \frac{15}{20}$

$\boxed{x = 41.41°}$

e)



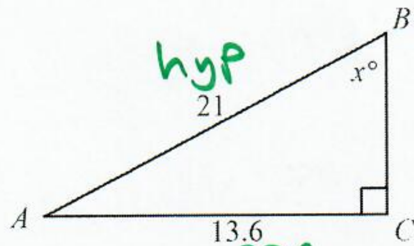
adj

opp

$\cancel{\tan^{-1}}\ \cancel{\tan}\ x = \tan^{-1} \frac{12}{13}$

$\boxed{x = 42.71°}$

f)



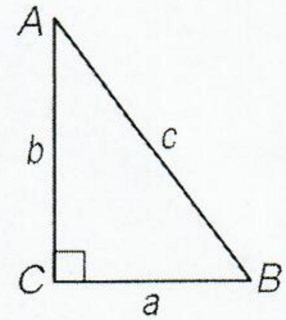
hyp

opp

$\cancel{\sin^{-1}}\ \cancel{\sin}\ x = \sin^{-1} \frac{13.6}{21}$

$\boxed{x = 40.36°}$

Solving a Triangle: Figuring out the lengths of all three sides and the measures of all three angles of a triangle.



★ If you know the lengths of two of the sides, use the Pythagorean Theorem to find the length of the third side.

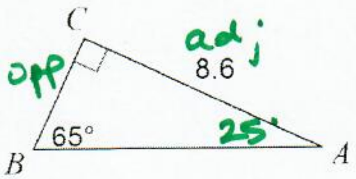
★ If you know the measure of one of the acute angles, use the fact that the angles in a triangle add to 180° to find the measure of the other angle.

★ If you know the measure of one angle and the length of one side, use sin, cos, or tan to figure out the lengths of the other sides.

★ If you know the lengths of the sides and need to figure out the angle measures, use inverse functions ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$).

Examples: ΔABC . Round answers to the nearest tenth. Show all your work.

a)



$m\angle A =$ 25°

$m\angle B =$ 65°

$m\angle C =$ 90°

$a =$ 4.0

$b =$ 8.6

$c =$ 9.5

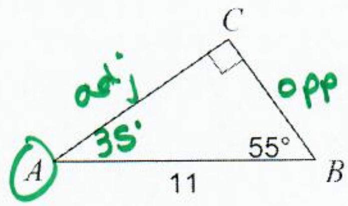
$8.6 \tan 25 = \frac{a \cdot \cancel{8.6}}{\cancel{8.6}} = 4.0$

$4^2 + 8.6^2 = c^2$

$\sqrt{89.96} = \sqrt{c^2}$

$9.5 = c$

b)



$m\angle A =$ 35°

$m\angle B =$ 55°

$m\angle C =$ 90°

$a =$ 6.3

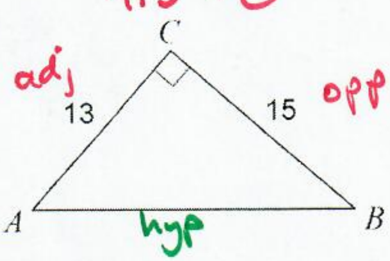
$b =$ 9

$c =$ 11

$11 \cdot \sin 35° = \frac{a \cdot \cancel{11}}{\cancel{11}} = 6.3$

$11 \cdot \cos 35° = \frac{b \cdot \cancel{11}}{\cancel{11}}$

c)



$m\angle A =$ 49.1°

$m\angle B =$ 40.9°

$m\angle C =$ 90°

$a =$ 15

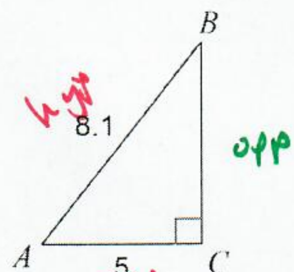
$b =$ 13

$c =$ 19.8

$\tan^{-1}$ ~~tan~~ $A = \tan^{-1}\frac{15}{13} = 49.1°$

$\frac{c \cdot \sin 49.1}{\sin 49.1} = \frac{15 \cdot \cancel{c}}{\cancel{c} \sin 49.1} = 19.8$

d)



$m\angle A =$ 51.9°

$m\angle B =$ 38.1°

$m\angle C =$ 90°

$a =$ 6.37

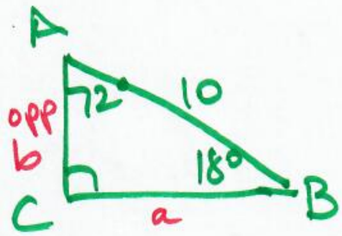
$b =$ 5

$c =$ 8.1

$\cos^{-1}$ ~~cos~~ $A = \cos^{-1}\frac{5}{8.1} = 51.9°$

$8.1 \cdot \sin 51.9 = \frac{a \cdot \cancel{8.1}}{\cancel{8.1}} = 6.37$

e) $m\angle A = 72°$, $c = 10$



$m\angle A = 72°$

$m\angle B = 18°$

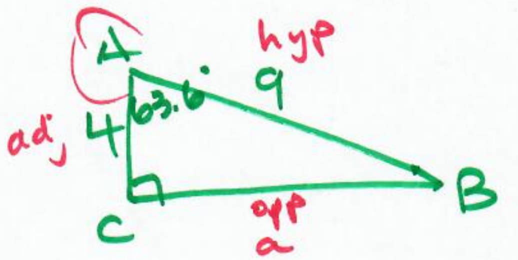
$m\angle C = 90°$

$a = 9.5$ $\quad 10 \cdot \cos 18 = \frac{a \cdot \cancel{10}}{\cancel{10}}$

$b = 25.7$ $\quad 10 \cdot \sin 18 = \frac{b \cdot \cancel{10}}{\cancel{10}}$

$c = 10$

f) $b = 4$, $c = 9$



$m\angle A = 63.6°$ $\quad \cos^{-1} \cancel{\cos} A = \frac{4}{9}$

$m\angle B = 26.4°$

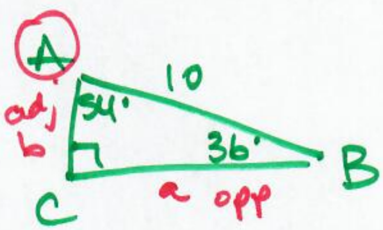
$m\angle C = 90°$

$a = 8.06$ $\quad 9 \cdot \sin 63.6° = \frac{a \cdot \cancel{9}}{\cancel{9}}$

$b = 4$

$c = 9$

g) $m\angle B = 36°, c = 10$



$m\angle A = 54°$

$m\angle B = 36°$

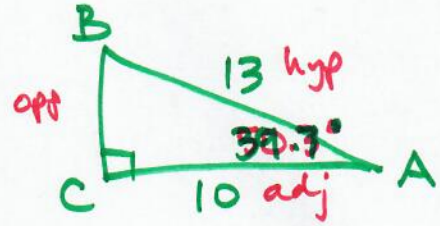
$m\angle C = 90°$

$a = 8.1$ $\quad 10 \sin 54 = \frac{a \cdot \cancel{10}}{\cancel{10}}$

$b = 5.9$ $\quad 10 \cos 54 = \frac{b \cdot \cancel{10}}{\cancel{10}}$

$c = 10$

h) $b = 10, c = 13$



$m\angle A = 39.7°$ $\quad \cos^{-1} \cancel{\cos} A = \frac{10}{13}$

$m\angle B = 50.3°$

$m\angle C = 90°$

$a = 8.3$ $\quad 13 \cdot \sin 39.7° = \frac{a \cdot \cancel{13}}{\cancel{13}}$

$b = 10$

$c = 13$

8.3 Trigonometry on the Cartesian Plane

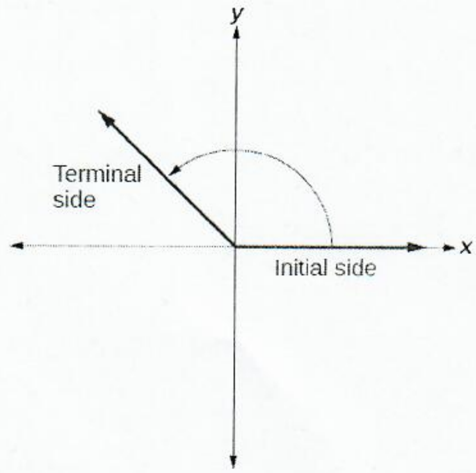
Cartesian Plane- is a **plane** with a rectangular coordinate system that associates each point in the **plane**with a [illegible] of numbers. We know this as the x and y axis.

Standard Position- the vertex of the angle is on the origin of the x and y axis and the angle is measured counterclockwise from the positive x-axis.

Terminal Side- the ray that makes the angle when its initial side is in standard position

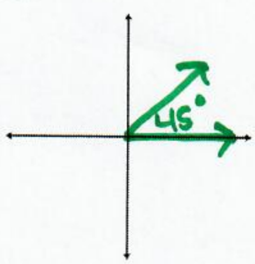
Reference angle-is the smallest **angle** that you can make from the terminal side of an **angle** with the x-axis. This angle measure will always be less than 90°.

Standard Position

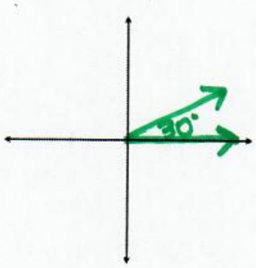


Example: Draw the angle measurement in standard position. Identify the location of the reference angle and its measure.

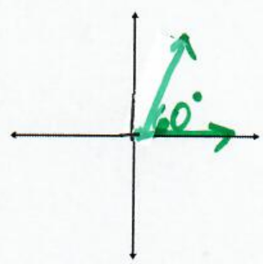
a. 45°



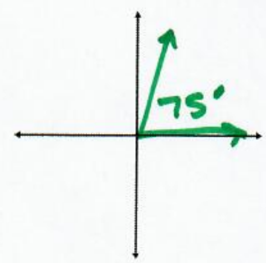
b. 30°



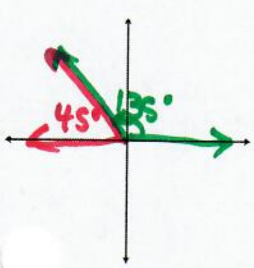
c. 60°



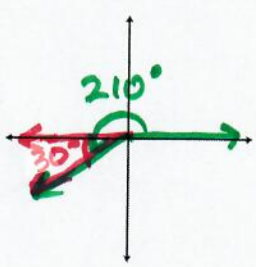
d. 75°



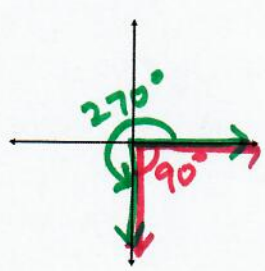
e. 135°



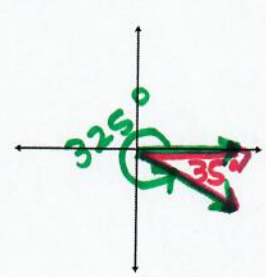
f. 210°



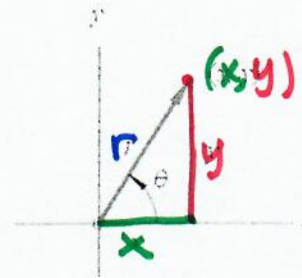
g. 270°



h. 325°



When we first defined the trigonometric functions the angle θ was between 0° and 90° and we used the terms *adjacent*, *opposite* and *hypotenuse* to refer to the sides of a triangle.



But we now want to allow angle θ to have values outside this range. These triangles can have an angle that is bigger than 90°.

To allow for angles bigger than 90° we now imagine an arrow pointing out from the origin with length r and orientated at angle θ, and with its terminal side ending at (x, y).

We construct a triangle by drawing a line vertically from the arrowhead to the x axis and another line horizontally across to the y axis.

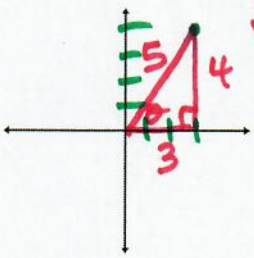
We now redefine the six trigonometric functions like this: $\boldsymbol{sin\theta = \frac{y}{r}}$ $\boldsymbol{cos\theta = \frac{x}{r}}$ $\boldsymbol{tan\theta = \frac{y}{x}}$

If we are given a coordinate, we will know the value of x and y, but how could you find the value of r?

__Example__: **Find the sine, cosine, and tangent of the following angles made by coordinate points. Keep answers in simplified radical form (NO DECIMALS!)**

a) $(3,4)$

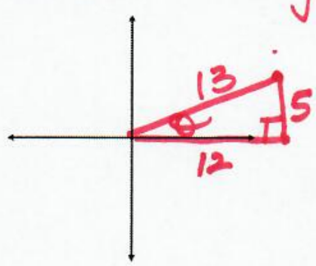
$3^2+4^2=r^2$
$9+16=r^2$
$\sqrt{25}=\sqrt{r^2}$
$5=r$



$\sin\theta = 4/5$
$\cos\theta = 3/5$
$\tan\theta = 4/3$

b) $(12,5)$

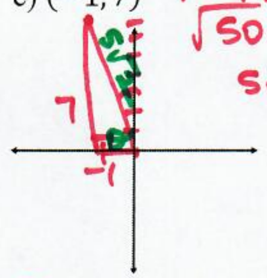
$12^2+5^2=r^2$
$144+25=r^2$
$\sqrt{169}=\sqrt{r^2}$
$13=r$



$\sin\theta = 5/13$
$\cos\theta = 12/13$
$\tan\theta = 5/12$

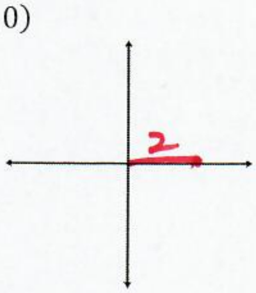
c) $(-1,7)$

$(-1)^2+7^2=r^2$
$1+49=r^2$
$\sqrt{50}=\sqrt{r^2}$
$5\sqrt{2}=r$



$\sin\theta = \frac{7}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{10}$
$\cos\theta = \frac{-1}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{10}$
$\tan\theta = \frac{7}{-1} = -7$

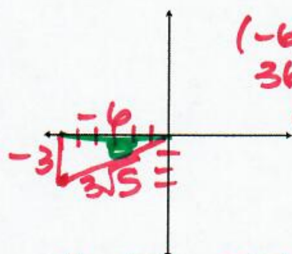
d) $(2,0)$



$\sin\theta = \frac{0}{2} = 0$
$\cos\theta = \frac{2}{2} = 1$
$\tan\theta = \frac{0}{2} = 0$

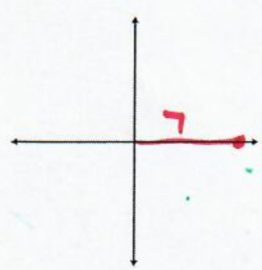
e) $(-6,-3)$

$(-6)^2+(-3)^2=r^2$
$36+9=r^2$
$\sqrt{45}=\sqrt{r^2}$
$3\sqrt{5}=r$



$\sin\theta = \frac{-3}{3\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{-3\sqrt{5}}{15} = -\frac{\sqrt{5}}{5}$
$\cos\theta = \frac{-6}{3\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{-6\sqrt{5}}{15} = \frac{-2\sqrt{5}}{5}$
$\tan\theta = \frac{-3}{-6} = \frac{1}{2}$

f) $(7,0)$



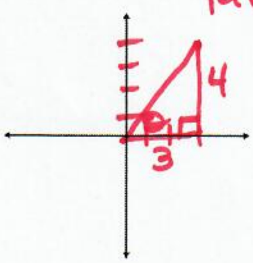
$\sin\theta = \frac{0}{7} = 0$
$\cos\theta = \frac{7}{7} = 1$
$\tan\theta = \frac{0}{7} = 0$

We can also use inverse trigonometric functions to find the angle created by points on the coordinate plane. Remember: <u>TrigFunction(angle/theta) = Ratio</u> so <u>Inverse function(ratio) = theta.</u>

Find the measurement of the STANDARD ANGLE (you will need to first find the reference angle!) that is created by the coordinate point. Draw a picture. Round to the ten-thousandths place. (4 decimal places)

a) $(3, 4)$

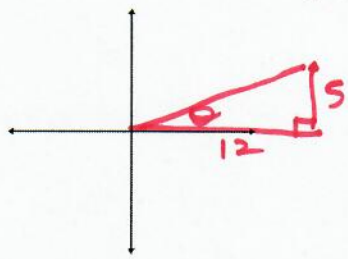
$\tan\theta = \frac{4}{3}$



$\theta =$ 53.1301°

b) $(12, 5)$

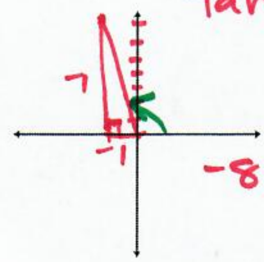
$\tan\theta = \frac{5}{12}$



$\theta =$ 22.6199°

c) $(-1, 7)$

$\tan\theta = \frac{7}{-1}$

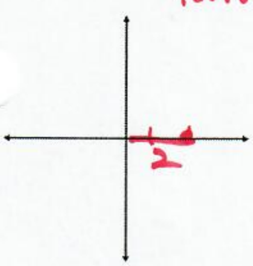


-81.8699

$\theta =$ 98.1301°

d) $(2, 0)$

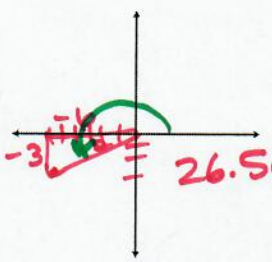
$\tan\theta = \frac{0}{2}$



$\theta =$ 0°

e) $(-6, -3)$

$\tan\theta = \frac{-3}{-6}$

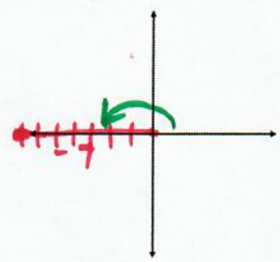


26.5651 + 180°

$\theta =$ 206.5651°

f) $(-7, 0)$

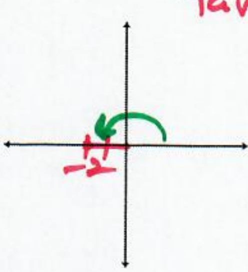
$\tan\theta = \frac{0}{-7}$



$\theta =$ 180°

g) $(-2, 0)$

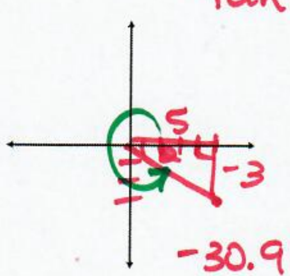
$\tan\theta = \frac{0}{-2}$



$\theta =$ 180°

h) $(5, -3)$

$\tan\theta = \frac{-3}{5}$

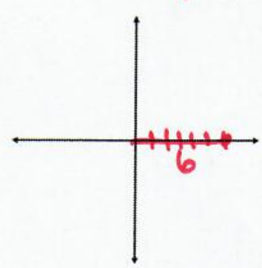


-30.9638°

$\theta =$ 329.0362°

i) $(6, 0)$

$\tan\theta = \frac{0}{6}$

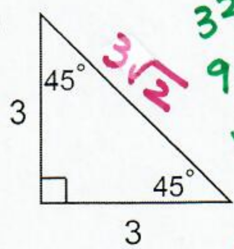


$\theta =$ 0°

8.4 Special Right Triangles

U‥e the Pythagorean Theorem to find the length of the hypotenuse for each right triangle. Express your ‥swers in simplest radical form.

a)



$3^2+3^2=c^2$

$9+9=c^2$

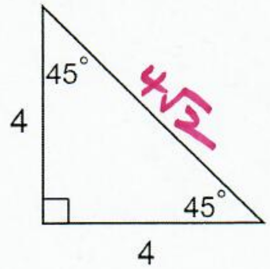
$\sqrt{18}=\sqrt{c^2}$

9 2

3 3

$\boxed{3\sqrt{2}=c}$

b)



$4^2+4^2=c^2$

$16+16=c^2$

$\sqrt{32}=\sqrt{c^2}$

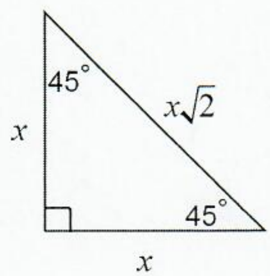
4 8

4 2

$\boxed{4\sqrt{2}=c}$

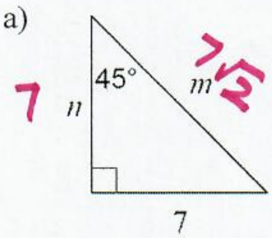
45°-45°-90° Right Triangles:

- **Legs are the same length**
- **Hypotenuse = Leg $\times\sqrt{2}$**

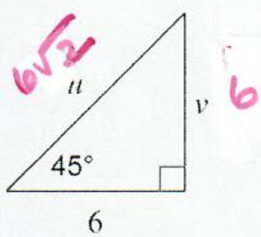


‥amples: Find the value of each variable.

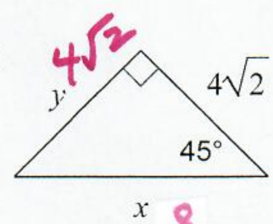
a)



b)



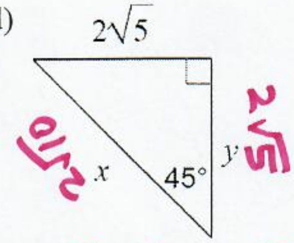
c)



$4\sqrt{2}\cdot\sqrt{2}=4\sqrt{4}$

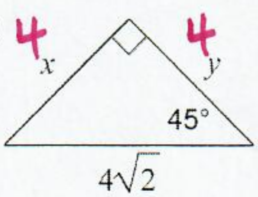
$=4\cdot 2=8$

d)

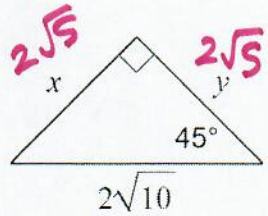


$2\sqrt{5}\cdot\sqrt{2}=2\sqrt{10}$

e)

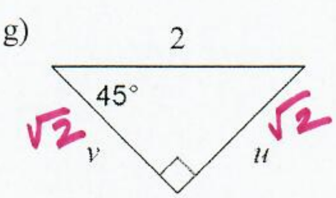


f)



$\frac{2\sqrt{10}}{\sqrt{2}}=2\sqrt{5}$

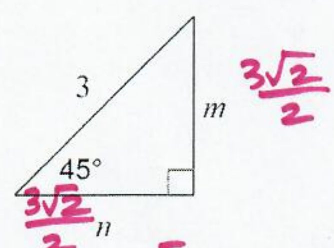
g)



$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}$

$=\sqrt{2}$

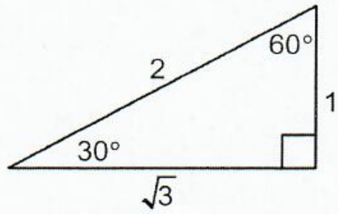
h)

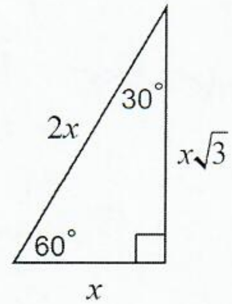


$\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$

30°-60°-90° Right Triangles:

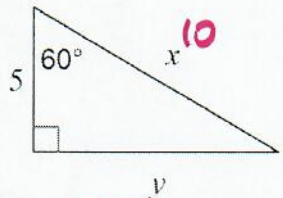
- **Hypotenuse = 2 × Short Leg**
- **Long Leg = Short Leg × $\sqrt{3}$**





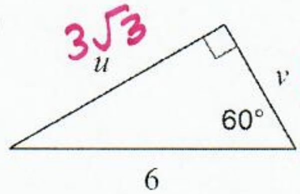
Examples: Find the value of each variable.

a)



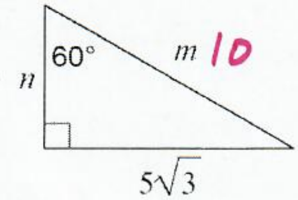
$x = 10$, $y = 5\sqrt{3}$

b)



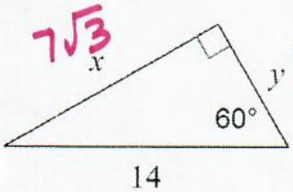
$u = 3\sqrt{3}$, $v = 3$

c)



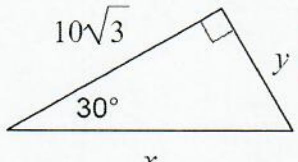
$n = 5$, $m = 10$

d)



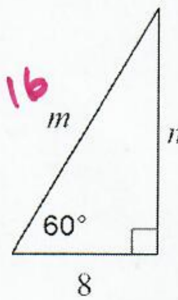
$x = 7\sqrt{3}$, $y = 7$

e)



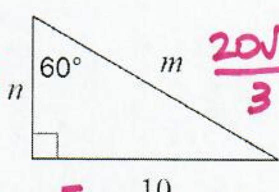
$y = 10$, $x = 20$

f)



$m = 16$, $n = 8\sqrt{3}$

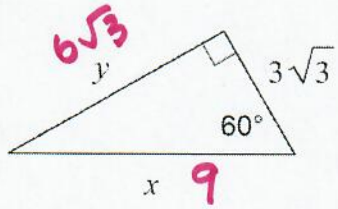
g)



$n = \frac{10\sqrt{3}}{3}$, $m = \frac{20\sqrt{3}}{3}$

$\frac{10}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$

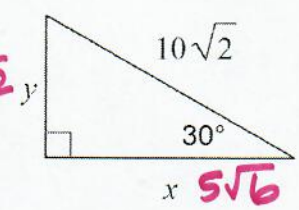
h)



$y = 6\sqrt{3}$, $x = 9$

$3\sqrt{3} \cdot \sqrt{3} = 3\sqrt{9} = 3 \cdot 3 = 9$

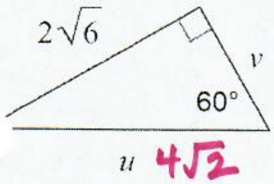
i)



$y = 5\sqrt{2}$, $x = 5\sqrt{6}$

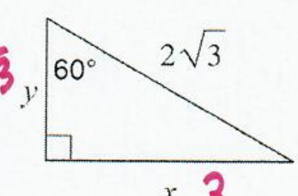
$5\sqrt{2} \cdot \sqrt{3} = 5\sqrt{6}$

j)



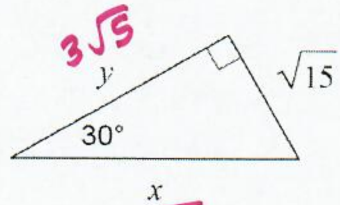
$v = 2\sqrt{2}$, $u = 4\sqrt{2}$

$\frac{2\sqrt{6}}{\sqrt{3}} = 2\sqrt{2}$



$y = \sqrt{3}$, $x = 3$

$\sqrt{3} \cdot \sqrt{3} = \sqrt{9} = 3$



$y = 3\sqrt{5}$, $x = 2\sqrt{15}$

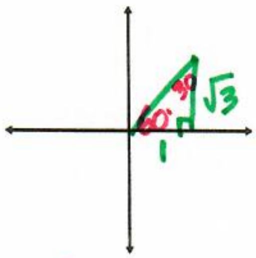
$\sqrt{15} \cdot \sqrt{3} = \sqrt{45} = 3\sqrt{5}$

3 3 9 5

8.4 Special Right Triangles (continued)

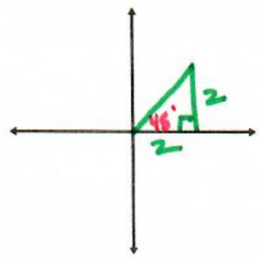
Find the measurement of the STANDARD ANGLE (you will need to first find the reference angle!) that is located by the coordinate point. Draw a picture. Use special right triangles to solve for the angle. NO DECIMAL ANSWERS ALLOWED!

A. $(1, \sqrt{3})$



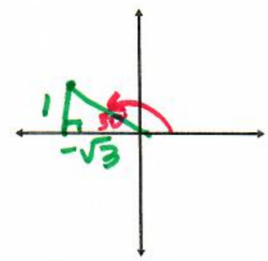
$\theta = 60°$

B. $(2,2)$



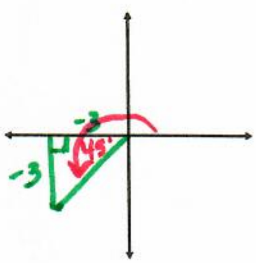
$\theta = 45°$

C. $(-\sqrt{3}, 1)$



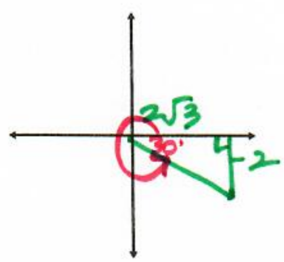
$\theta = 180° - 30° = 150°$

D. $(-3, -3)$



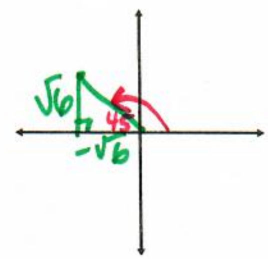
$\theta = 180° + 45° = 225°$

E. $(2\sqrt{3}, -2)$



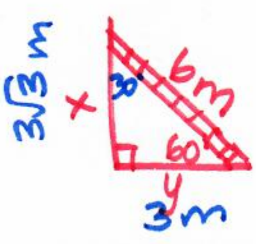
$\theta = 360° - 30° = 330°$

F. $(-\sqrt{6}, \sqrt{6})$



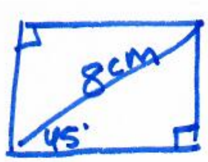
$\theta = 180° - 45° = 135°$

A six-meter-long ladder leans against a building. If the ladder makes an angle of 60° with the ground, how far up the wall does the ladder reach? How far from the wall is the base of the ladder?



The ladder reaches $3\sqrt{3}$ meters up the wall.
The base of the ladder is 3 m. from the wall.

A square has a diagonal of length 8 cm. Find the length of each side.

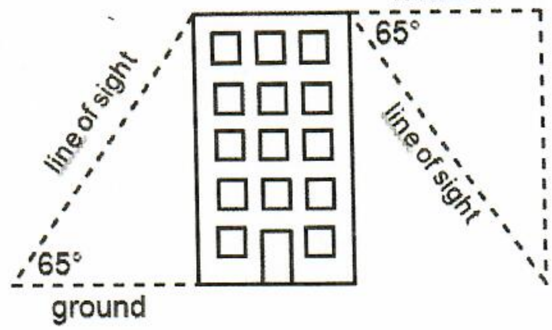


$\frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = \boxed{4\sqrt{2} \text{ cm}}$

8.5 Applications of Trigonometry

Angle of elevation- is the angle made with the ground and your line of sight to an object above you.

Angle of depression- is the angle from the horizon and your line of sight to an object below you.



Compass Bearing- is the direction towards which you are headed, as shown by a compass. It is most commonly written in the form N 6° E, meaning the bearing that makes an angle of 6° with North towards East. (North or South is usually given before East or West, and the angle never exceeds 90°.)

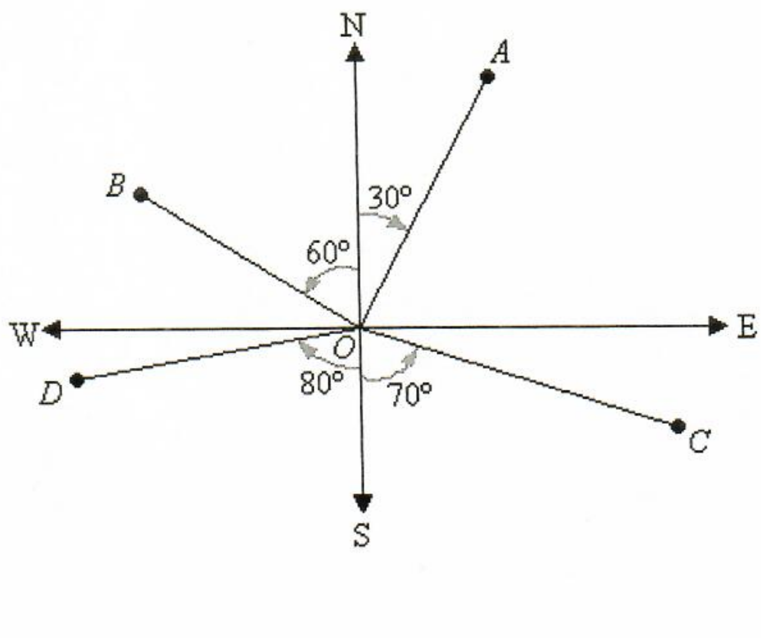
Write the correct compass bearing for the following points.

A. N 30° E

B. N 60° W

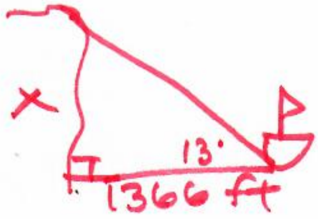
C. S 70° E

D. S 80° W



Draw a diagram for each of the following problems. Then write an equation to represent the situation and then solve the problem. Round your answers to the nearest tenth.

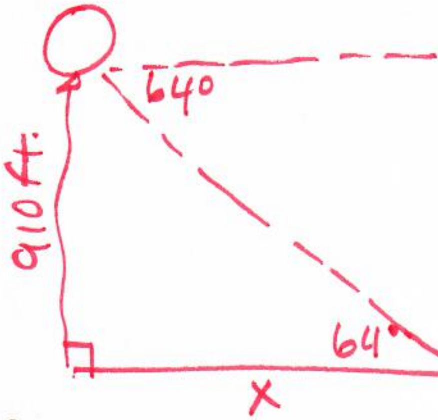
Example 1:
From a boat on the lake, the angle of elevation to the top of a cliff is 13°. If the base of the cliff is 1366 feet from the boat, how high is the cliff?



$$1366 \cdot \tan 13° = \frac{x}{1366} \cdot 1366$$

$x = \boxed{315.37 \text{ feet}}$

Example 2:
From a balloon 910 feet high, the angle of depression to the ranger headquarters is 64°. How far is the headquarters from a point on the ground directly below the balloon?

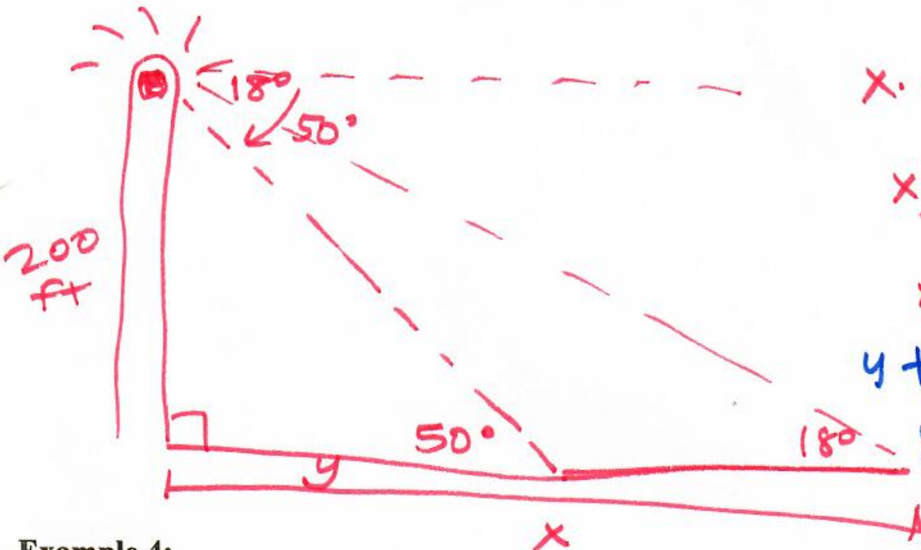


$x \cdot \tan 64^\circ = \frac{910}{\cancel{x}} \cdot \cancel{x}$

$\frac{x \cancel{\tan 64^\circ}}{\cancel{\tan 64^\circ}} = \frac{910}{\tan 64^\circ}$

$x = \boxed{443.84 \text{ feet}}$

Example 3:
A person is watching a boat from the top of lighthouse. The boat is approaching the lighthouse directly. When first noticed, the angle of depression is 18°. When the boat stops, the angle of depression is 50°. The lighthouse is 200 feet tall. How far did the boat travel from when it was first noticed until it stopped?



$x \cdot \tan 18^\circ = \frac{200}{\cancel{x}} \cdot \cancel{x}$

$\frac{x \cancel{\tan 18^\circ}}{\cancel{\tan 18}} = \frac{200}{\tan 18}$

$x = 615.54 \text{ ft}$

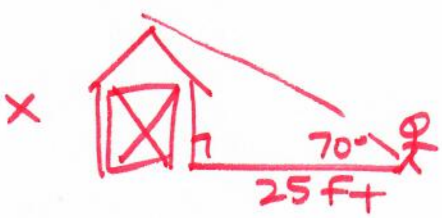
$y \tan 50^\circ = \frac{200}{\cancel{y}} \cdot \cancel{y}$

$\frac{y \cancel{\tan 50^\circ}}{\cancel{\tan 50^\circ}} = \frac{200}{\tan 50}$

$y = 167.82 \text{ ft}$

$615.54 - 167.82 = \boxed{447.72 \text{ ft}}$

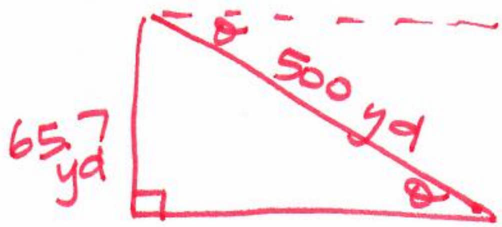
Example 4:
A person is 25 feet from the base of a barn. The angle of elevation from the level ground to the top of the barn is 70°. How tall is the barn?



$25 \cdot \tan 70^\circ = \frac{x}{\cancel{25}} \cdot \cancel{25}$

$\boxed{68.69 \text{ feet}}$

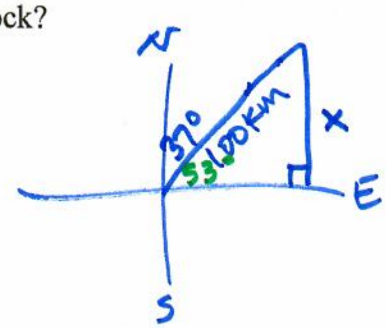
Example 5:
A sledding run is 500 yards long with a vertical drop of 65.7 yards. Find the angle of depression of the run.



$\sin^{-1}$ $\cancel{\sin^{-1}}\cancel{\sin}\,\theta = \frac{65.7}{500}$

$\boxed{\theta = 7.55^\circ}$

Example 6: A sailboat leaves the dock at a bearing of $N\ 37^\circ\ E$ and travels a distance of 100 km. Immediately after, the boat turns and travels due south. How far does the boat need to travel in order to be due east of the dock?



$100 \cdot \sin 53^\circ = \frac{x \cdot \cancel{100}}{\cancel{100}}$

$\boxed{x = 79.86 \text{ km}}$

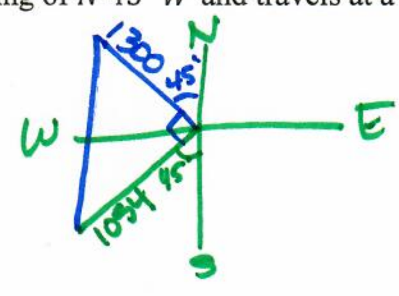
Example 7: Two planes leave from the airport at the same time. Plane A travels at a bearing of $S\ 45^\circ\ W$ and travels at a speed of 527 mph. Plane B travels at a bearing of $N\ 45^\circ\ W$ and travels at a speed of 650 mph.

After 2 hours how far apart are the planes?

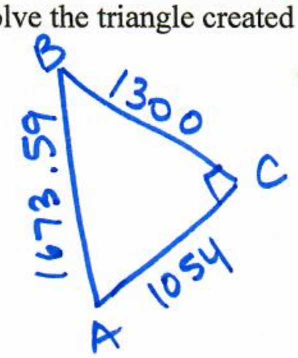
$1054^2 + 1300^2 = c^2$

$2800916 = c^2$

$\boxed{1673.59 = c \text{ miles}}$



Solve the triangle created by the two planes and the airport after the 2 hours of travel?



$\sin^{-1}$ $\cancel{\sin^{-1}}\cancel{\sin}\,A = \frac{1300}{1673.59}$

$A = 50.97^\circ$ $a = 1300$ miles

$B = 90 - 50.97 = 39.03^\circ$ $b = 1054$ miles

$C = 90^\circ$ $c = 1673.59$ miles