SM2H

Secondary Math 2 Honors
Unit 4 Graphing Quadratic Functions

4.0 Forms of Quadratic Functions

standard **Form:** $f(x) = ax^2 + bx + c$, where $a \neq 0$. There are no parentheses.
Example: $f(x) = -3x^2 + 2x - 7$

factored **Form:** $f(x) = a(x-p)(x-q)$, where $a \neq 0$. Written as a multiplication problem.
Also known as **intercept form**.
Example: $f(x) = (x-4)(x+5)$

Vertex **Form:** $f(x) = a(x-h)^2 + k$, where $a \neq 0$. x only shows up once, as part of a perfect square.
Example: $f(x) = 2(x+7)^2 - 1$

Conic Form of a parabola: $4p(y-k) = (x-h)^2$ or $4p(x-h) = (y-k)^2$
Examples: $4(y-2) = (x+5)^2$ or $-8(x+6) = (y-1)^2$

Examples: State whether each quadratic function is in standard, factored, or vertex form.

a) $f(x) = 2(x+3)(x-5)$
factored

b) $f(x) = -(x+4)^2 - 5$
Vertex

c) $f(x) = x^2 + 2x + 4$
Standard

d) $f(x) = -x^2 + 5x$
standard

e) $f(x) = 3x(x-2)$
factored

f) $f(x) = 2(x+1)^2 - 3$
vertex

g) $f(x) = -(x+5)^2$
Vertex

h) $f(x) = -3x^2 + 4$
standard or Vertex

i) $f(x) = 5x^2$
standard or vertex

4.1 Graphing Quadratic Functions: Vertex and Axis of Symmetry

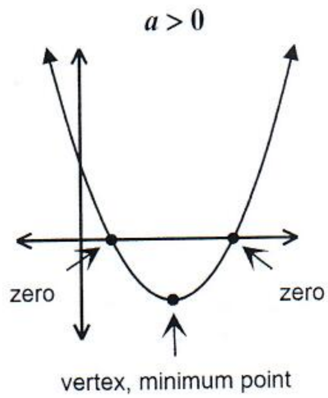
Vocabulary:

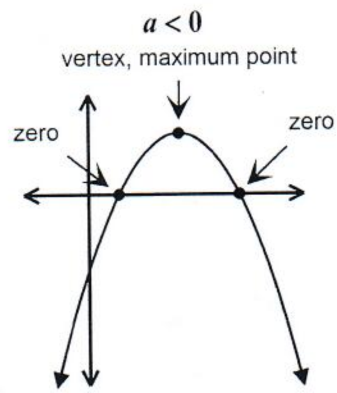
parabola: The shape of the graph of a quadratic function.

axis of symmetry: A line that cuts a parabola in half. If you were to fold a parabola along its axis of symmetry, the two sides would overlap. The equation of the axis of symmetry looks like $x = \#$.

vertex: The "tip" of the parabola – the point at which it changes direction.

- If the parabola opens up ($a > 0$), the vertex is the lowest point on the graph, or the ***minimum point.***
- If the parabola opens down ($a < 0$), the vertex is the highest point on the graph, or the ***maximum point.***





Finding the y-intercept:

1. Plug in 0 for x.
2. Simplify. Don't forget order of operations.

Vertex Form of a Quadratic Function: $y = a(x-h)^2 + k$

Vertex: (h,k)

Axis of Symmetry: $x = h$

- The sign of h is the *opposite* of the sign in the equation. h moves the graph of $y = x^2$ right and left in the *opposite* direction as the sign in the equation (but the *same* direction as the sign of h itself).
- The sign of k is the *same* as the sign in the equation. k moves the graph of $y = x^2$ up and down in the *same* direction as the sign in the equation.
    - For $y = (x-2)^2 + 5$, $h = 2$ and $k = 5$. The vertex is $(2,5)$ and the axis of symmetry is $x = 2$. The graph of $y = x^2$ moved *right* 2 and *up* 5.
    - For $y = (x+3)^2 - 7$, $h = -3$ and $k = -7$. The vertex is $(-3,-7)$ and the axis of symmetry is $x = -3$. The graph of $y = x^2$ moved *left* 3 and *down* 7.

Direction of Opening:

- Opens up if a is positive.
- Opens down if a is negative.

Examples: For each function, do the following: 1) State the coordinates of the vertex. 2) State the direction of the opening, that is, whether the parabola opens up or down. 3) Find the y-intercept. 4) Draw a rough sketch of the graph. 5) Find the Domain and Range

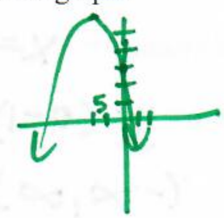
a) $y=-3(x+2)^2+27$ Vertex: $(-2, 27)$ Direction: down

axis of symmetry: $x=-2$

y-intercept:

$y=-3(0+2)^2+27$
$=-3(2)^2+27$
$=-12+27$
$=15$
$(0, 15)$

sketch graph:



Domain: $(-\infty, \infty)$ Range: $(-\infty, 27]$

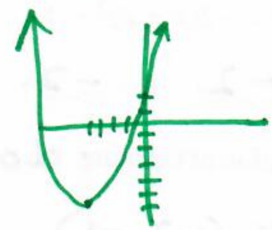
c) $f(x)=\frac{1}{2}(x+4)^2-6$ Vertex: $(-4, -6)$ Direction: up

axis of symmetry: $x=-4$

y-intercept:

$y=\frac{1}{2}(0+4)^2-6$
$=\frac{1}{2}(4)^2-6$
$=8-6$
$=2$
$(0, 2)$

sketch graph:



Domain: $(-\infty, \infty)$ Range: $[-6, \infty)$

Vertical Stretch:

- a changes how wide or narrow the graph is.
  - If $|a|>1$, the graph is *narrower* than the graph of $y=x^2$.
  - If $|a|<1$, the graph is *wider* than the graph of $y=x^2$.
- Figure out the exact shape of the graph by making an x,y table. Always use the vertex as one point. Then choose two x-values on each side of the vertex to plug into the equation to find the corresponding y-coordinates.
- A shortcut is to use counting patterns to graph the parabola. Start at the vertex, then count:

  $\leftrightarrow 1, \uparrow a$
  $\leftrightarrow 2, \uparrow 4a$ If a is negative, count down instead of up.
  $\leftrightarrow 3, \uparrow 9a$, etc.

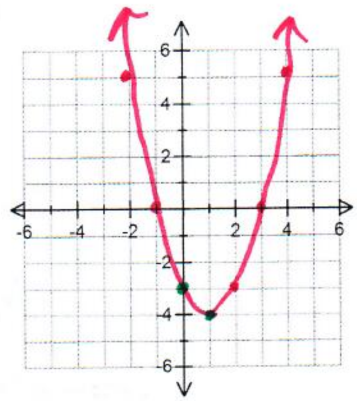
  - For $y=2(x-3)^2-4,\ a=2.$

    Start at the vertex $(3,-4)$, and count $\leftrightarrow 1, \uparrow 2; \leftrightarrow 2, \uparrow 8; \leftrightarrow 3, \uparrow 18\ldots$

$f(x) = a(x-h)^2 + k$

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = (x-1)^2 - 4$



$\leftrightarrow 1, \uparrow 1a$

$\leftrightarrow 2, \uparrow 4a$

$\leftrightarrow 3, \uparrow 9a$

$a =$ 1 $\quad h =$ 1 $\quad k =$ -4

Direction of Opening: up

Vertex: $(1, -4)$

Is it a maximum or minimum point? minimum

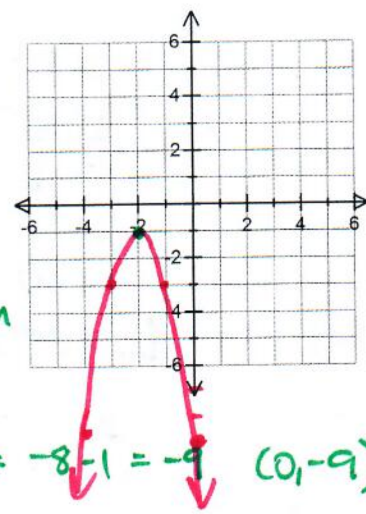
Axis of Symmetry: $x = 1$

y-intercept: $y = (0-1)^2 - 4 = (-1)^2 - 4 = -3 \quad (0, -3)$

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

b) $f(x) = -2(x+2)^2 - 1$



$a =$ -2 $\quad h =$ -2 $\quad k =$ -1

Direction of Opening: down

Vertex: $(-2, -1)$

Is it a maximum or minimum point? maximum

Axis of Symmetry: $x = -2$

y-intercept: $y = -2(0+2)^2 - 1 = -2(2)^2 - 1 = -8 - 1 = -9 \quad (0, -9)$

Domain: $(-\infty, \infty)$

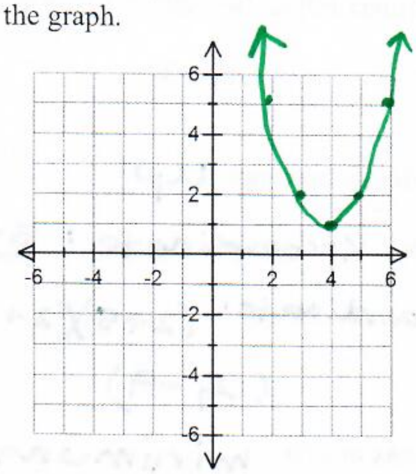
Range: $(-\infty, -1]$

Standard Form: $f(x) = ax^2 + bx + c$

- Just like with the other forms, the graph opens up if a is positive and opens down if a is negative.
- **Vertex:**
  - The x-coordinate of the vertex is $\frac{-b}{2a}$. (The opposite of b divided by 2 times a)
  - To find the y-coordinate, plug the x-coordinate into the original equation.

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = x^2 - 8x + 17$ $a =$ 1 $b =$ -8 $c =$ 17



Direction of Opening: up

Vertex: x-coordinate $= -\frac{b}{2a} = \frac{8}{2} = 4$

y-coordinate: $4^2 - 8(4) + 17 = 16 - 32 + 17 = 1$

$(4, 1)$

Is it a maximum or minimum point? minimum

What is the maximum/minimum **value**? 1

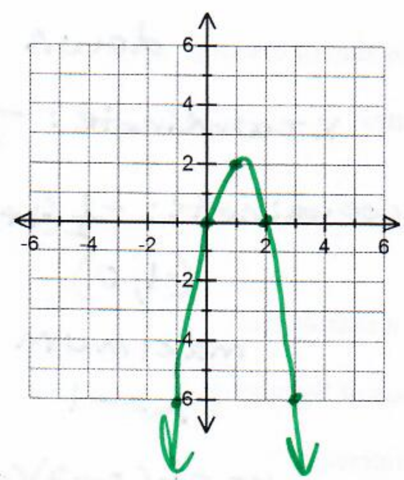
Axis of Symmetry: $x = 4$

y-intercept: $y = 0^2 - 8(0) + 17 = 17$ $(0, 17)$

Domain: $(-\infty, \infty)$

Range: $[1, \infty)$

b) $y = -2x^2 + 4x$ $a =$ -2 $b =$ 4 $c =$ 0



Direction of Opening: down

Vertex: x-coordinate $= -\frac{b}{2a} = \frac{-4}{2(-2)} = \frac{-4}{-4} = 1$

y-coordinate: $-2(1)^2 + 4(1) = -2 + 4 = 2$

$(1, 2)$

Is it a maximum or minimum point? maximum

What is the maximum/minimum **value**? 2

Axis of Symmetry: $x = 1$

y-intercept:

$y = -2(0) + 4(0) = 0$ $(0, 0)$

Factored Form: $f(x) = a(x - p)(y - q)$

- Like other forms, a is the vertical stretch
- $(p, 0)$ and $(q, 0)$ are the x-intercepts (zeroes). The x-value of the vertex is exactly half-way between them
- Evaluate the function at $x = \frac{p+q}{2}$ to find the y value of the vertex.

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = (x-5)(x+1)$ $\qquad$ $p =$ 5 $\qquad$ $q =$ -1

direction of opening: up

vertex: x-coordinate : $\frac{5+-1}{2} = \frac{4}{2} = 2$

y-coordinate: $(2-5)(2+1) = (-3)(3) = -9$

$(2, -9)$

Is it max or min: minimum

Axis of Symmetry: $x = 2$

y-intercept: $y = (0-5)(0+1) = (-5)(1) = -5$

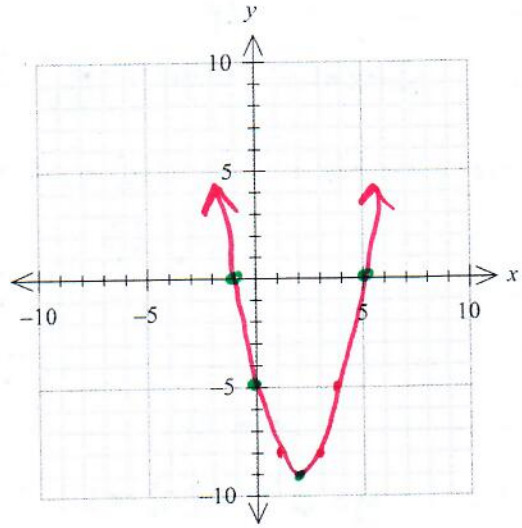
$(0, -5)$

Domain:

$(-\infty, \infty)$

Range:

$[-9, \infty)$



b) $y = -\frac{1}{2}(x+3)(x-1)$ $\qquad$ $p =$ -3 $\qquad$ $q =$ 1

direction of opening: down

vertex: x-coordinate : $\frac{-3+1}{2} = \frac{-2}{2} = -1$

y-coordinate: $-\frac{1}{2}(-1+3)(-1-1) = -\frac{1}{2}(2)(-2) = 2$

$(-1, 2)$

Is it max or min:
maximum

Axis of Symmetry: $x = -1$

y-intercept: $y = -\frac{1}{2}(0+3)(0-1) = -\frac{1}{2}(3)(-1) = \frac{3}{2}$

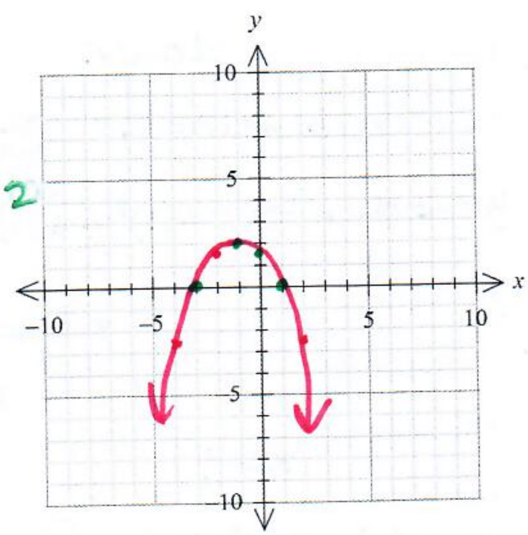
$(0, 3/2)$

Domain:

$(-\infty, \infty)$

Range:

$(-\infty, 2]$



4.2 Graphing using zeroes, solutions, roots, and x-intercepts

Zeros *of a Function:* The values of x that make $f(x)$ or y equal zero. If the zeros are real, they tell you the places where the graph crosses the x-axis, or the ***x*-intercepts** of the graph.

Other words for zeros: ***solutions to*** $f(x)=0$, ***roots, x-intercepts.***

Finding zeros (x-intercepts):

1. Change y or $f(x)$ to 0.
2. Solve for x.
    - If the equation is in factored form, solving for x is easy – just think "What would x have to be to make each set of parentheses equal to 0?"
    - If the equation is in standard form, solve by factoring or by using quadratic formula
    - If the equation is in vertex form, get the perfect square by itself, take the square root of both sides (don't forget the ±), then solve for x.

★ If your answers are imaginary (negative under the square root), the graph doesn't have x-intercepts.

Examples: For each equation, find the zeros and state whether the graph opens up or down. Then match the equation to the correct graph.

a) $y=(x-1)(x+3)$

zeros: 1, -3

b) $f(x)=-\frac{1}{2}x(x+2)$

zeros: 0, -2

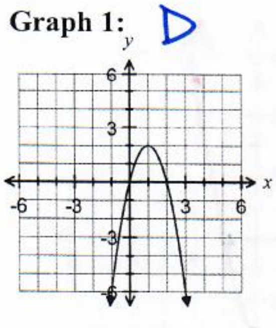
c) $y=3(x-3)(x-1)$

zeros: 3, 1

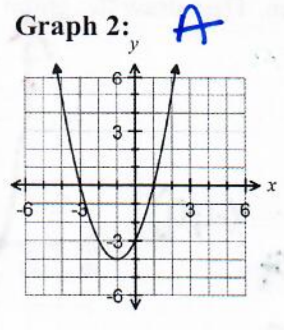
d) $f(x)=-2x(x-2)$

zeros: 0, 2

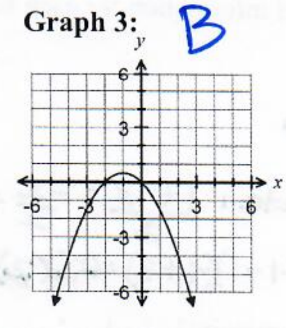
Graph 1: D



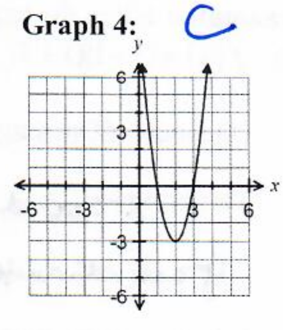
Graph 2: A



Graph 3: B



Graph 4: C



Examples: For each function, do the following: 1) State whether the parabola has a maximum or minimum. 2) State whether the parabola opens up or down. 3) Find the x-intercept(s). 4) Find the y-intercept. 5) Draw a rough sketch of the graph.

a) $f(x) = -(x+4)(x-1)$

Max/Min: maximum

Direction: down

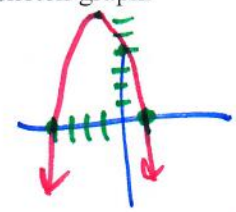
x-intercept(s):

(−4, 0)
(1, 0)

y-intercept:

$y = -(0+4)(0-1)$
$= -(4)(-1)$
$= 4$
$(0, 4)$

sketch graph:



vertex: x-coordinate $= \frac{-4+1}{2}$
$= -\frac{3}{2}$

y-coordinate:
$-(-3/2+4)(-3/2-1) = -(\frac{5}{2})(-\frac{5}{2})$
$= \frac{25}{4}$

$(-3/2, 25/4)$

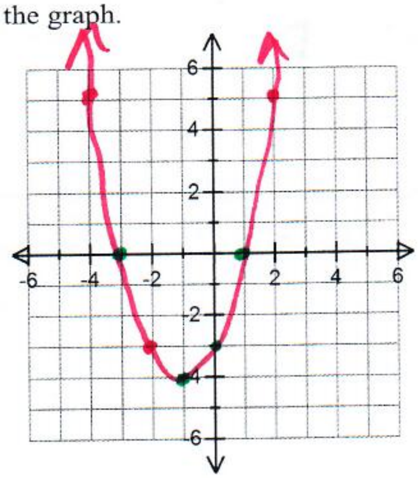
Graphing from Factored Form:

1. Determine whether the parabola will open up or open down.
2. Find the zeros or x-intercepts.
   - Let $f(x) = 0$ and solve the equation.
   - Mark the x-intercepts on the graph.
3. Find the y-intercept but substituting in $x = 0$. Mark this point on the graph.
4. Find the axis of symmetry and the vertex. Use the method from 4.1
   - If there is only one zero, then the axis of symmetry will run vertically through that point and that x-intercept will also be the vertex.
5. Use the pattern from Section 4.1 to find other points on the graph now that you have the location of the vertex.

**Remember that a parabola is a smooth curve. Do not draw straight lines!

Examples: Fill in the requested information for each function. Then draw the graph.

a) $f(x) = (x-1)(x+3)$



Direction of Opening: up

Vertex: x-coordinate: $\frac{1+-3}{2} = \frac{-2}{2} = -1$ $(-1, -4)$

y-coordinate: $(-1-1)(-1+3) = (-2)(2) = -4$

Is it a maximum or minimum point? minimum

Axis of Symmetry: $x = -1$

Zeros (x-intercepts): 1, −3 (1, 0); (−3, 0)

y-intercept:

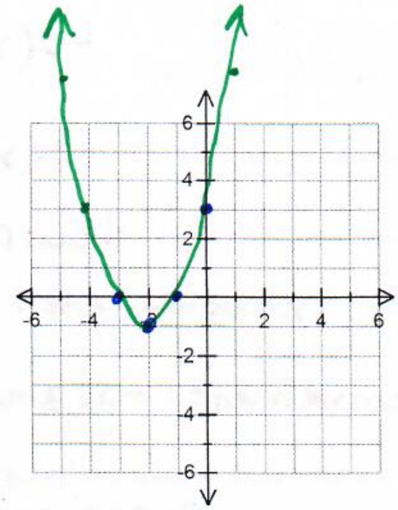
$y = (0-1)(0+3)$
$(-1)(3) = -3$
$(0, -3)$

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

b) $f(x) = x^2 + 4x + 3$

3·1



Factored Form: $f(x) =$ (x+3)(x+1)

Direction of Opening: up

Vertex: x-coordinate: $\frac{-3+-1}{2} = -\frac{4}{2} = -2$

y-coordinate: $(-2+3)(-2+1) = (1)(-1) = -1$

Is it a maximum or minimum point? minimum $(-2,-1)$

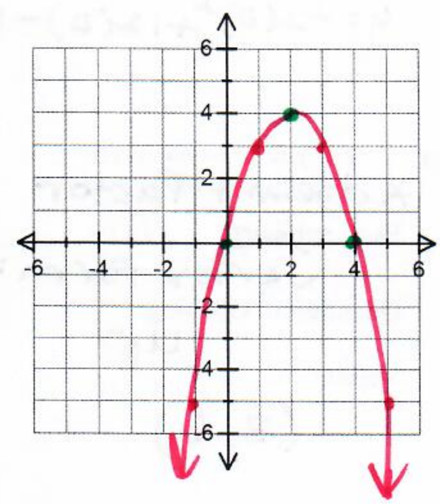
Axis of Symmetry: $x = -2$

Zeros (x-intercepts): $-3, -1$ $(-3,0)$ $(-1,0)$

y-intercept:

$y = 0^2 + 4(0) + 3 = 3$ $(0,3)$

c) $f(x) = -x^2 + 4x$



Factored Form: $f(x) =$ $-x(x-4)$

Direction of Opening: down

Vertex: x-coordinate: $\frac{0+4}{2} = \frac{4}{2} = 2$

y-coordinate: $-2(2-4) = -2(-2) = 4$

Is it a maximum or minimum point? maximum $(2,4)$

Axis of Symmetry: $x = 2$

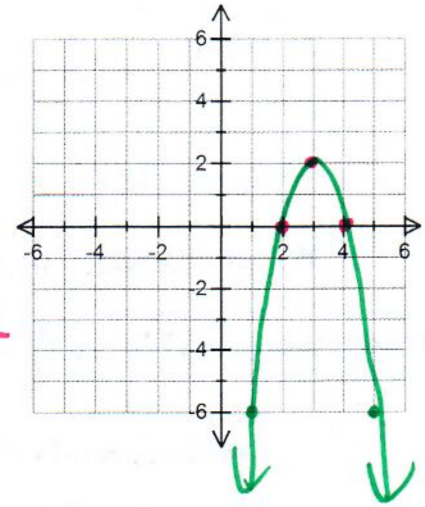
Zeros (x-intercepts):

0, 4

y-intercept:

$y = -0^2 + 4(0) = 0$ $(0,0)$

d) $f(x) = -2x^2 + 12x - 16$ $\quad -2(x^2 - 6x + 8)$ $\quad -4 \cdot -2$



Factored Form: $f(x) = -2(x-4)(x-2)$

Direction of Opening: down

Vertex: x-coordinate: $\frac{2+4}{2} = \frac{6}{2} = 3$

y-coordinate: $-2(3-4)(3-2) = -2(-1)(1) = 2$

$(3, 2)$

Is it a maximum or minimum point?

maximum

Axis of Symmetry:

$x = 3$

Zeros (x-intercepts):

4, 2 $\quad (4, 0)$ $\quad (2, 0)$

y-intercept:

$y = -2(0)^2 + 12(0) - 16 = -16 \quad (0, -16)$

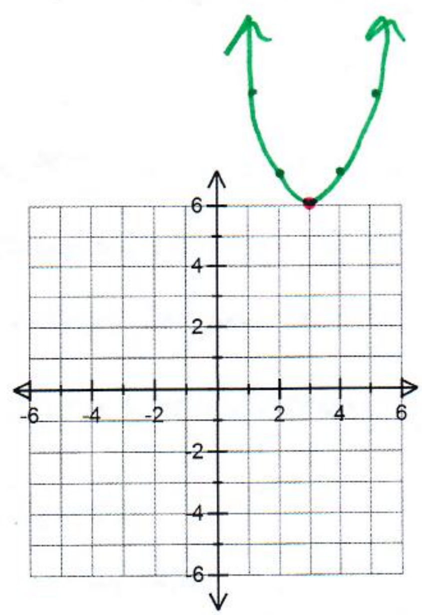
e) $f(x) = x^2 - 6x + 15$ $\quad y - 15 = x^2 - 6x + 9$

* doesn't factor

$y - \cancel{6} = (x-3)^2 + 6$

Factored ~~Form~~: $f(x) =$ ________

Vertex form: $y = (x-3)^2 + 6$



Direction of Opening:

up

Vertex:

$(3, 6)$

Is it a maximum or minimum point? minimum

Axis of Symmetry: $x = 3$

Zeros (x-intercepts):

none

$0 = (x-3)^2 + \cancel{6}$

$-6 \qquad -\cancel{6}$

$\sqrt{-6} = \sqrt{(x+3)^2}$

$\pm i\sqrt{6} = x + 3$

$\rightarrow$ $x = \boxed{-3 \pm i\sqrt{6}}$

y-intercept:

$y = 0^2 - 6(0) + 15$

$= 15 \quad (0, 15)$

4.3 Writing equations from a graph or from set of information

Examples: Write a quadratic equation or function in **Vertex Form**: $f(x) = y = a(x-h)^2 + k$

If you know the vertex of a parabola, (h,k), then you still need at least one other point on the parabola in order to write an equation.

- Use the vertex form and fill in all the information you have.
- Then use the point on the parabola and substitute in for x and y.
- Solve for a.
- Write your final equation

a) Vertex: $(2,1)$, passes through $(4,13)$

$y = a(x-2)^2 + 1$

$13 = a(4-2)^2 + 1$

$13 = a(2)^2 + \cancel{1}$

$-1 \quad \cancel{-1}$

$\frac{12}{4} = \frac{4a}{4}$

$3 = a$

$f(x) = 3(x-2)^2 + 1$

b) Vertex: $(-5,3)$, passes through $(-1,-29)$

$y = a(x+5)^2 + 3$

$-29 = a(-1+5)^2 + 3$

$-29 = a(4)^2 + 3$

$-29 = 16a + \cancel{3}$

$-3 \quad \cancel{-3}$

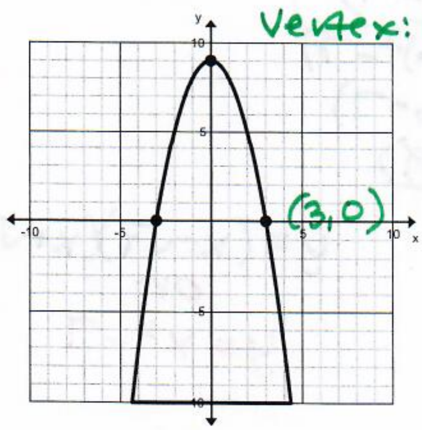
$\frac{-32}{16} = \frac{16a}{16}$

$-2 = a$

$f(x) = -2(x+5)^2 + 3$

Examples: Write the equation of each parabola based on the information in the graph. Follow the steps outlined above. Leave the equations in **Vertex Form**.

a) Vertex: $(0,9)$



$y = a(x-0)^2 + 9$

$0 = a(3-0)^2 + 9$

$0 = a(3)^2 + 9$

$0 = 9a + \cancel{9}$

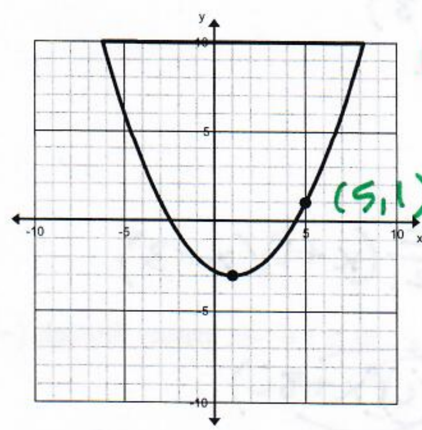
$-9 \quad \cancel{-9}$

$\frac{-9}{9} = \frac{9a}{9}$

$-1 = a$

$f(x) = -x^2 + 9$

Vertex: $(1,-3)$



$y = a(x-1)^2 - 3$

$1 = a(5-1)^2 - 3$

$1 = a(4)^2 - \cancel{3}$

$+3 \quad \cancel{+3}$

$\frac{4}{16} = \frac{16a}{16}$

$\frac{1}{4} = a$

$f(x) = \frac{1}{4}(x-1)^2 - 3$

Examples: Write a quadratic equation or function in **Factored Form**: $f(x) = y = a(x-p)(x-q)$

Use the factored form if you know the roots (a.k.a. solutions, x-intercepts, zeros). You will still need to know at least one other point on the parabola in order to write an equation.

- Use the factored form and fill in all the information you have.
- Then use the point on the parabola and substitute in for x and y.
- Solve for a.
- Write your final equation

a) Roots: $(3,0)\,\&\,(-2,0)$, goes through $(2,-4)$

$y = a(x-3)(x+2)$

$-4 = a(2-3)(2+2)$

$-4 = a(-1)(4)$

$\frac{-4}{-4} = \frac{a(\cancel{-4})}{\cancel{-4}}$

$1 = a$

$f(x) = (x-3)(x+2)$

b) x-intercept: $(3,0)$ & $(-2,0)$, goes through $(0,12)$

$y = a(x-3)(x+2)$

$12 = a(0-3)(0+2)$

$12 = a(-3)(2)$

$\frac{12}{-6} = \frac{a(\cancel{-6})}{\cancel{-6}}$

$-2 = a$

$f(x) = -2(x-3)(x+2)$

c) Zeros: $x=-1$ & $x=3$, goes through $(6,-10)$

$y = a(x+1)(x-3)$

$-10 = a(6+1)(6-3)$

$-10 = a(7)(3)$

$\frac{-10}{21} = \frac{a(\cancel{21})}{\cancel{21}}$

$-\frac{10}{21} = a$

$f(x) = -\frac{10}{21}(x+1)(x-3)$

d) Roots: $x=\sqrt{7}$ & $x=-\sqrt{7}$, goes through $(-6,29)$

$y = a(x-\sqrt{7})(x+\sqrt{7})$

$y = a(x^2 + \cancel{\sqrt{7}x} - \cancel{\sqrt{7}x} - \sqrt{49})$

$\boxed{y = a(x^2-7)}$

$29 = a((-6)^2-7)$

$29 = a(36-7)$

$\frac{29}{29} = \frac{a(\cancel{29})}{\cancel{29}}$

$1 = a$

$y = (x-\sqrt{7})(x+\sqrt{7})$

or

$y = x^2 - 7$

e) Solutions: $x=8i$ & $x=-8i$, passes through $(-2,-204)$

$y = a(x-8i)(x+8i)$

$y = a(x^2 + \cancel{8i} - \cancel{8i} - 64i^2)$ (-1)

$\boxed{y = a(x^2+64)}$

$-204 = a((-2)^2+64)$

$\frac{-204}{68} = \frac{a(\cancel{68})}{\cancel{68}}$

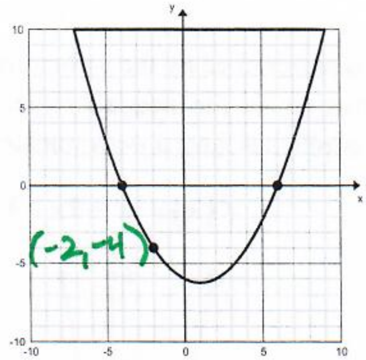
$-3 = a$

$f(x) = -3(x^2+64)$

$f(x) = -3(x-8i)(x+8i)$ OR $f(x) = -3x^2 - 192$

Examples: Write the equation of each parabola based on the information in the graph. Leave the equations in **Factored Form**.

a)



$y = a(x+4)(x-6)$

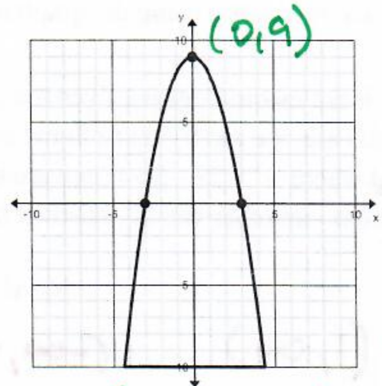
$-4 = a(-2+4)(-2-6)$

$-4 = a(2)(-8)$

$\frac{-4}{-16} = \frac{a(-16)}{-16}$

$\frac{1}{4} = a \quad f(x) = \frac{1}{4}(x+4)(x-6)$

b)



$y = a(x+3)(x-3)$

$9 = a(0+3)(0-3)$

$9 = a(3)(-3)$

$\frac{9}{-9} = \frac{a(-9)}{-9}$

$-1 = a \quad f(x) = -(x+3)(x-3)$

Examples: Write a quadratic equation or function in **Standard Form**: $f(x) = ax^2 + bx + c$

- First write the equation in either Vertex Form or Factored Form (whichever seems easier)
- Then use correct order of operations to multiply/distribute in order to get rid of parenthesis
- List the three terms in correct order: x^2, then x, then the constant term

a) Use the graph just above to write the equation in **Standard Form**

$f(x) = \frac{1}{4}(x+4)(x-6)$

$= \frac{1}{4}(x^2 - 6x + 4x - 24)$

$= \frac{1}{4}(x^2 - 2x - 24)$

$f(x) = \frac{1}{4}x^2 - \frac{1}{2}x - 6$

b) Write one of the vertex form equations in **Standard Form**

$f(x) = 3(x-2)^2 + 1$

$= 3(x-2)(x-2) + 1$

$= 3(x^2 - 2x - 2x + 4) + 1$

$= 3(x^2 - 4x + 4) + 1$

$= 3x^2 - 12x + 12 + 1$

$f(x) = 3x^2 - 12x + 13$

c) Write the equation with the following characteristics in **Factored Form**, **Vertex Form** and **Standard Form** (yes, we'll need to complete the square to get it into vertex form ☺)

roots at $(-3, 0)$ and $(5, 0)$ and goes through the point $(2, -15)$

$y = a(x+3)(x-5)$

$-15 = a(2+3)(2-5)$

$-15 = a(5)(-3)$

$\frac{-15}{-15} = \frac{a(-15)}{-15}$

$1 = a$

factored:

$f(x) = (x+3)(x-5)$

standard:

$f(x) = (x+3)(x-5)$

$= x^2 - 5x + 3x - 15$

$f(x) = x^2 - 2x - 15$

vertex:

$f(x) = x^2 - 2x - 15$

$y + 15 = x^2 - 2x + 1$
$\quad +1 \qquad\qquad -1$

$y + 16 = (x-1)^2$
$\quad -16 \qquad\quad -16$

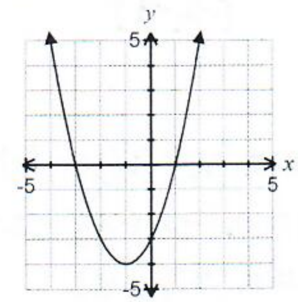
$f(x) = (x-1)^2 - 16$

4.4 Solving Quadratic Inequalities & Systems of Equations by Graphing

Examples: Solve each inequality using the graph of $f(x) = x^2 + 2x - 3$.

Notice that each of these inequalities involves the value of $x^2 + 2x - 3$, which is represented by the y-coordinate of the graph. In each case, we are trying to figure out what x-values (x-coordinates) make the inequality true. When trying to find where $x^2 + 2x - 3 > 0$, we are trying to figure out what x-coordinates have a y-coordinate that is bigger than zero—in other words, *where is the graph above the x-axis?*

$f(x) = x^2 + 2x - 3$



a) $x^2 + 2x - 3 > 0$

$(-\infty, -3) \cup (1, \infty)$

b) $x^2 + 2x - 3 \geq 0$

$(-\infty, -3] \cup [1, \infty)$

c) $x^2 + 2x - 3 < 0$

$(-3, 1)$

d) $x^2 + 2x - 3 \leq 0$

$[-3, 1]$

Solving a Quadratic Inequality Using the Graph:

1. Write the inequality in standard form. Replace the inequality sign with an equal sign and solve the equation $ax^2 + bx + c = 0$ by factoring, completing the square, or using the quadratic formula. This gives you the x-intercepts of the graph of $y = ax^2 + bx + c$.

2. Graph $y = ax^2 + bx + c$. The graph does not have to be very detailed. A rough sketch of a parabola opening in the correct direction with the correct x-intercepts is all you need.

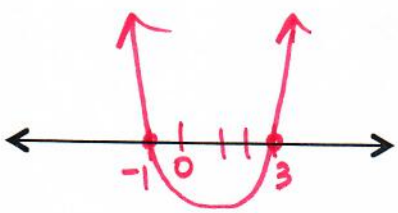
3. The solutions of $ax^2 + bx + c > 0$ are the x-values for which the graph is **above** the x-axis.
   The solutions of $ax^2 + bx + c \geq 0$ are the x-values for which the graph is **on or above** the x-axis.
   The solutions of $ax^2 + bx + c < 0$ are the x-values for which the graph is **below** the x-axis.
   The solutions of $ax^2 + bx + c \leq 0$ are the x-values for which the graph is **on or below** the x-axis.

4. If the inequality involves $\leq$ or $\geq$, the x-intercepts **are included** in the solution set (use brackets).
   If the inequality involves $<$ or $>$, the x-intercepts **are not included** in the solution set (use parentheses).

Examples: Solve each quadratic inequality and write the solution set in interval notation.

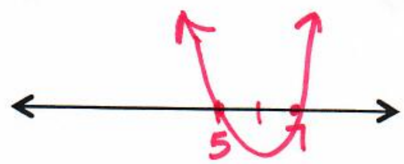
a) $(x-3)(x+1) \ge 0$

$(-\infty,-1] \cup [3,\infty)$



b) $(x-7)(x-5) < 0$

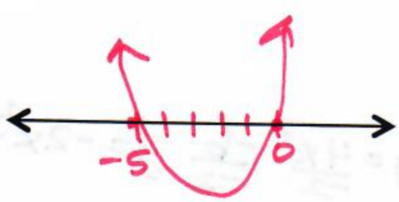
$(5,7)$



c) $x^2 + 5x > 0$

$x(x+5) > 0$

$(-\infty,-5) \cup (0,\infty)$

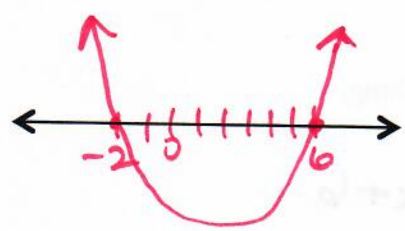


d) $x^2 - 4x - 12 \le 0$

$-6 \cdot 2$

$(x-6)(x+2) \le 0$

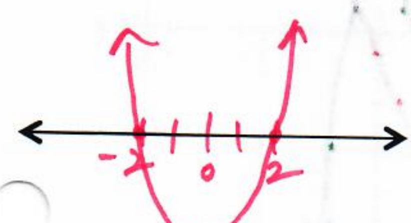
$[-2,6]$



e) $x^2 - 4 < 0$

$(x+2)(x-2) < 0$

$(-2,2)$



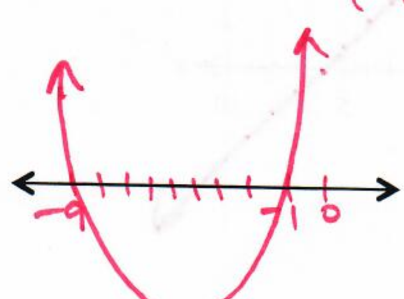
f) $x^2 + 10x \ge -9$

$x^2+10x+9 \ge 0$

$1 \cdot 9$

$(x+1)(x+9) \ge 0$

$(-\infty,-9] \cup [-1,\infty)$



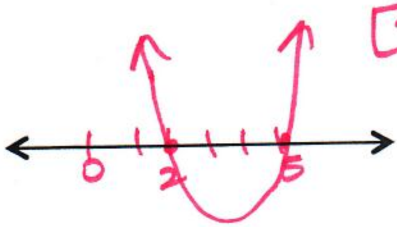
g) $x^2 + 10 \le 7x$

$x^2 - 7x + 10 \le 0$

$-2 \cdot -5$

$(x-2)(x-5) \le 0$

$[2,5]$



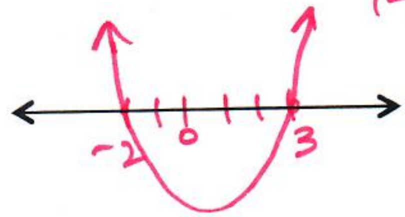
h) $x^2 > x + 6$

$x^2 - x - 6 > 0$

$-3 \cdot 2$

$(x-3)(x+2) > 0$

$(-\infty,-2) \cup (3,\infty)$



Solving Systems of Equations by Graphing

Solving a system of equations means finding the values of x and y that make both equations true. The solutions are usually written as ordered pairs (x, y).

Solving by graphing:

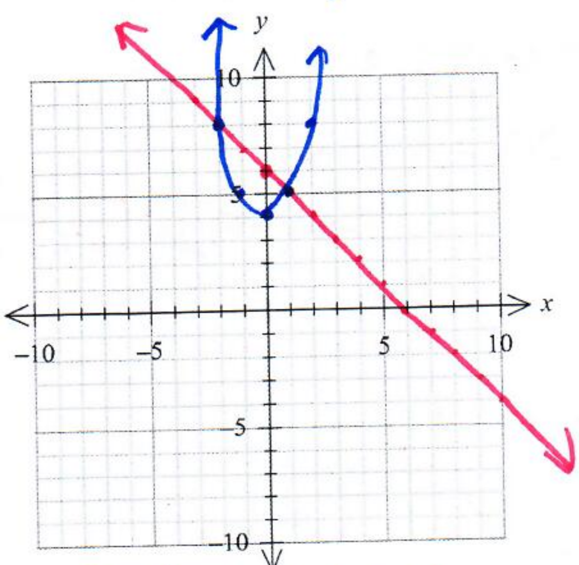
1. Solve both equations for y.
2. Graph both equations using $y = mx + b$, transformations, or x, y tables.
3. The points where the two graphs intersect (cross) are the solutions.
4. Write the solutions as ordered pairs.
5.

Examples: Solve by graphing.

a) $x + y = 6$ $\quad y = -x + 6$

$y = x^2 + 4$

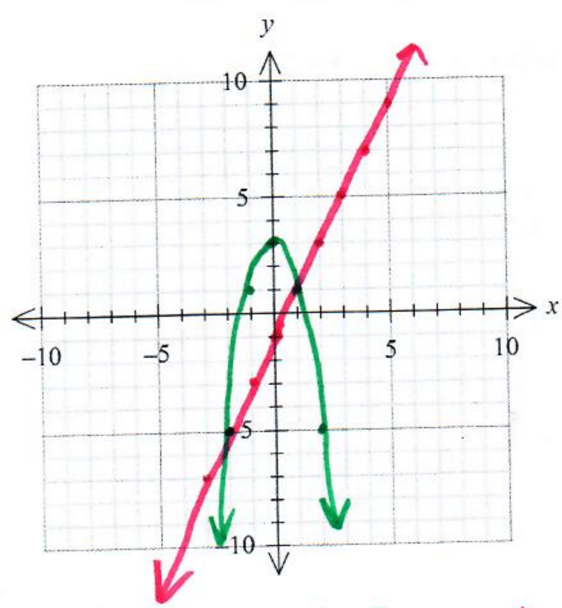
vertex: $(0,4)$



$(1,5)$ and $(-2,8)$

b) $6 - 2y = 4x^2$ $\quad \frac{-2y}{-2} = \frac{4x^2}{-2} - \frac{6}{-2}$ $\quad y = -2x^2 + 3$

$6x - 3y = 3$ $\quad \frac{-3y}{-3} = \frac{-6x}{-3} + \frac{3}{-3}$ $\quad y = 2x - 1$



$(1,1)$ and $(-2,-5)$

Vertex Form $y=a(x-h)^2+k$ $y=(x+5)^2-4$	**Standard Form** $y=ax^2+bx+c$ $y=x^2+10x-21$	**Factored Form** $y=a(x-p)(x-q)$ $y=(x+7)(x-3)$
	Finding the Vertex	
The vertex is easy to find: (opposite of # with x, # at the end) (h,k)	To find the x-value of the vertex, use $x=\frac{-b}{2a}$. Once you know the x-value, plug that # into the original equation to find the y-value.	Figure out where the zeros are (easy to do in factored form). The x-value of the vertex will be right in the middle of the zeros. To find the y-value of the vertex, plug the x-value into the original equation.
	Axis of Symmetry	
x = x-value of the vertex	x = x-value of the vertex	x = x-value of the vertex
	Direction of Opening	
Opens down if the # in front of the parentheses is negative. Otherwise, opens up.	Opens down if the # in front of x^2 is negative. Otherwise, opens up.	Opens down if the # in front of the parentheses is negative. Otherwise, opens up.
	***y*-intercept**	
Plug in zero for x, solve for y.	Plug in zero for x, solve for y.	Plug in zero for x, solve for y.
	Zeros (*x*-intercepts)	
1. Plug in zero for y. 2. Get perfect square by itself. 3. Take the $\sqrt{\ }$ of both sides. (Don't forget the ±!) 4. Solve for x.	1. Plug in zero for y. 2. Solve for x by factoring or by using the quadratic formula.	1. Plug in zero for y. 2. Finding the x-values is easy – Think "What would x have to be to make each set of parentheses equal 0?" The zeros are always the opposites of the numbers with x in the parentheses.

Domain: $(-\infty,\infty)$

Range: Look at the graph – (lowest y, highest y) – use a bracket on the maximum or minimum value!