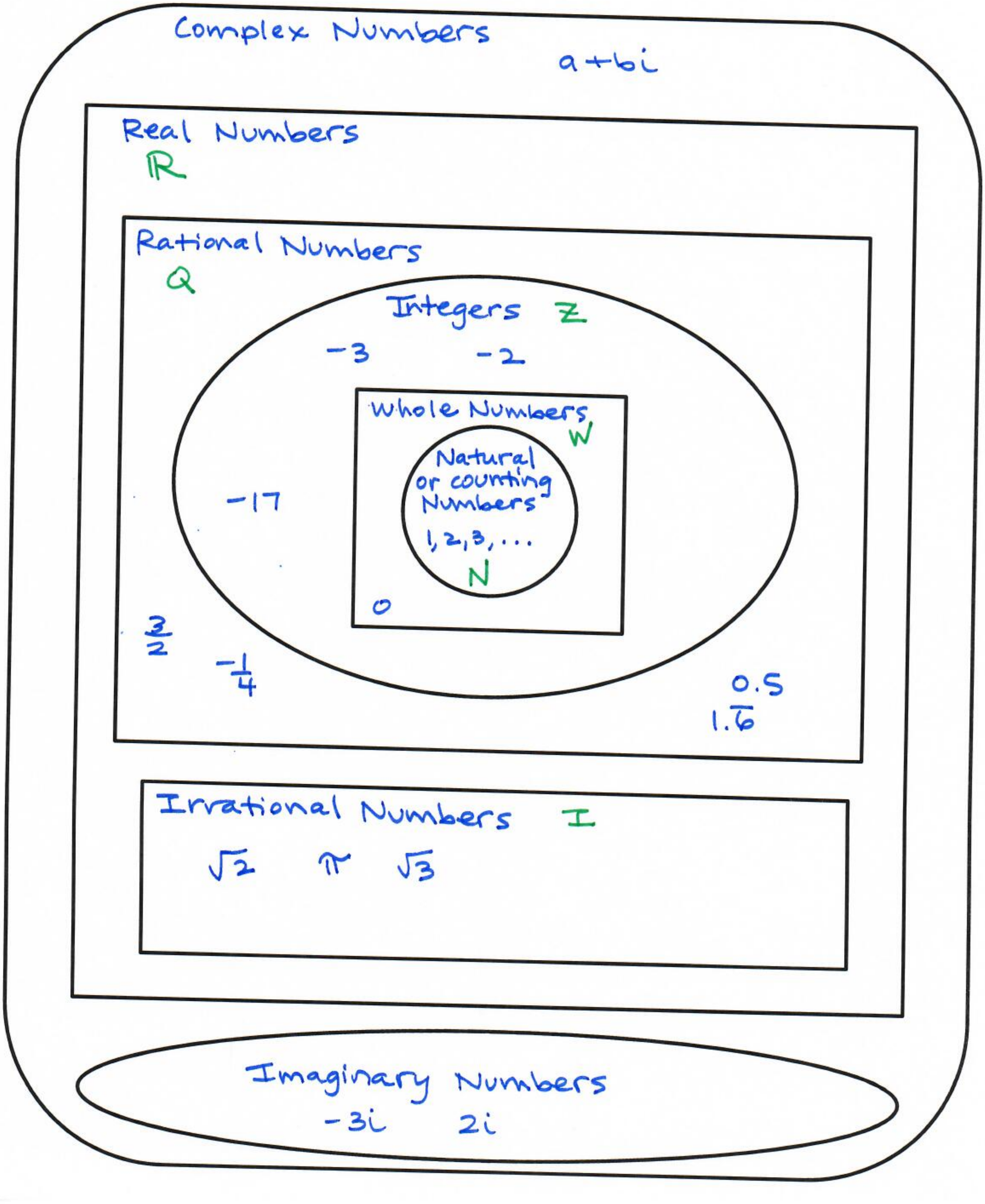
SM2H

Notes

Unit 2 – Polynomials, Exponents, Radicals & Complex Numbers

2.1 Number Theory



2.1 Adding, Subtracting, and Multiplying Polynomials Notes

Monomial: An expression that is a number, a variable, or numbers and variables multiplied together. Monomials only have variables with whole number exponents and never have variables in the denominator of a fraction or variables under roots.

Monomials: $5b,\ \frac{xyz}{8},\ -w,\ 23,\ x^2,\ \frac{1}{3}x^3y^4$ **Not Monomials:** $\frac{1}{x^4},\ \sqrt[3]{x},\ a^{-1},\ z^{\frac{1}{5}}$

Constant: A monomial that contains no variables, like 23 or -1.

Coefficient: The numerical part of a monomial (the number being multiplied by the variables.)

→ 5b 5 is the coefficient

Polynomial: A monomial or several monomials joined by + or – signs.

Terms: The monomials that make up a polynomial. Terms are separated by + or – signs.

Like Terms: Terms whose variables and exponents are exactly the same.

Binomial: A polynomial with two unlike terms.

$x-3$

Trinomial: A polynomial with three unlike terms.

x^2-5x+6

Adding and Subtracting Polynomials

To add or subtract polynomials, combine like terms. Add or subtract the coefficients. The variables and exponents do not change. ***Remember to subtract everything inside the parentheses after a minus sign.*** Subtract means "add the opposite," so change the minus sign to a plus sign and then change the signs of all the terms inside the parentheses.

* write answers in descending order

Examples: Simplify each expression.

a) $(5n^2-2)+(7-3n^2)$

$=\boxed{2n^2+5}$

b) $(4x^2-3x+1)+(-2x^2+5x-6)$

$=\boxed{2x^2+2x-5}$

c) $(2w^2+3w)-(4w^2+w)$

$= 2w^2+3w-4w^2-w$

$=\boxed{-2w^2+2w}$

d) $(-6x^2-3x+2)-(-4x^2-x+3)$

$= -6x^2-3x+2+4x^2+x-3$

$=\boxed{-2x^2-2x-1}$

i) $(6m^2+5m)-(4m^2-2m)+(3m^2-7m)$

$6m^2+5m-4m^2+2m+3m^2-7m$

$=\boxed{5m^2}$

j) $(-2k+5)+(k^2-3k)-(-4k^2+8)$

$= -2k+5+k^2-3k+4k^2-8$

$=\boxed{5k^2-5k-3}$

Multiplying Polynomials

To multiply two polynomials, distribute each term of one polynomial to each term of the other polynomial. Then combine any like terms. When you are multiplying two binomials, this is sometimes called the ***FOIL Method*** because you multiply **F** the *first* terms, **O** the *outside* terms, **I** the *inside* terms, and **L** the *last* terms.

Examples: Multiply.

a) $-xy\left(7x^2y+3xy-11\right)$

$= -7x^3y^2-3x^2y^2+11xy$

b) $(m+3)(m-8)$

$= m^2-8m+3m-24$

$= \boxed{m^2-5m-24}$

c) $(3x+1)(5x-2)$

$= 15x^2-6x+5x-2$

$= \boxed{15x^2-x-2}$

d) $\left(2u^2-1\right)\left(-5u^2+4\right)$

$= -10u^4+8u^2+5u^2-4$

$= \boxed{-10u^4+13u^2-4}$

e) $(2x-3)^2$

$= (2x-3)(2x-3)$

$= 4x^2-6x-6x+9$

$= \boxed{4x^2-12x+9}$

f) $(n+3)(n-3)$

$= n^2-3n+3n-9$

$= \boxed{n^2-9}$

g) $(2x-3)\left(5x^2-6x+7\right)$

$= 10x^3-12x^2+14x$
$\quad -15x^2+18x-21$

$= \boxed{10x^2-27x^2+32x-21}$

h) $\left(4x^2+7x-3\right)\left(x^2-2x+8\right)$

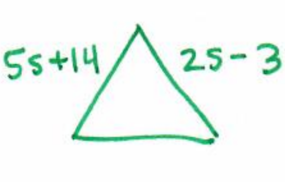
$= 4x^4-8x^3+32x^2$
$\quad +7x^3-14x^2+56x$
$\quad -3x^2+6x-24$

$= \boxed{4x^4-x^3+15x^2+62x-24}$

Perimeter

Perimeter = sum of all the sides

The measure of the perimeter of a triangle is 23s + 56. It is known that two of the sides of the triangle have measures of 2s - 3 and 5s + 14. Find the length of the third side.



$P = 23s+56$

$23s+56-(5s+14)-(2s-3)$

$= 23s+56-5s-14-2s+3$

$= \boxed{16s+45}$

Area = L·W

2.2 Rules of Exponents

The following properties are true for all real numbers a and b and all integers m and n, provided that no denominators are 0 and that 0^0 is not considered.

1 as an exponent: $a^1 = a$ e.g.) $7^1 = 7,\ \pi^1 = \pi,\ (-10)^1 = -10$

0 as an exponent: $a^0 = 1$ e.g.) $2^0 = 1,\ 27^0 = 1,\ \left(-\frac{5}{8}\right)^0 = 1$

The Product Rule: $a^m \cdot a^n = a^{m+n}$ e.g.) $x^2 \cdot x^5 = x^{2+5} = x^7$

The Quotient Rule: $\dfrac{a^m}{a^n} = a^{m-n}$ e.g.) $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

The Power Rule: $(a^m)^n = a^{mn}$ e.g.) $(x^2)^5 = x^{(2)(5)} = x^{10}$

Raising a product to a power: $(ab)^n = a^n b^n$ e.g.) $(2k)^4 = 2^4 \cdot k^4 = 16k^4$

Raising a quotient to a power: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ e.g.) $\left(\dfrac{p}{q^2}\right)^3 = \dfrac{p^3}{(q^2)^3} = \dfrac{p^3}{q^6}$

Negative exponents: $a^{-n} = \dfrac{1}{a^n}$ e.g.) $2^{-3} = \dfrac{1}{2^3},\ 7x^3y^{-4} = \dfrac{7x^3}{y^4}$

$\dfrac{1}{a^{-n}} = a^n$ e.g.) $\dfrac{1}{x^{-9}} = x^9,\ \dfrac{b}{c^{-3}d} = \dfrac{bc^3}{d}$

$\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}$ e.g.) $\left(\dfrac{2}{v}\right)^{-3} = \left(\dfrac{v}{2}\right)^3 = \dfrac{v^3}{2^3} = \dfrac{v^3}{8}$

Rational exponents: $a^{n/d} = \sqrt[d]{a^n}$ e.g.) $2^{\frac{3}{5}} = \sqrt[5]{2^3}$

To *simplify* an expression containing powers means to rewrite the expression without parentheses or negative exponents.

Examples: Simplify the following expressions.

a) $m^5 \cdot m^7$

$\boxed{m^{12}}$

b) $(5a^2b^3)(3a^4b^5)$

$\boxed{15a^6b^8}$

c) $\frac{r^9}{r^3}$

$\boxed{r^6}$

d) $\frac{p^3}{p^7} = p^{-4}$

$= \boxed{\frac{1}{p^4}}$

e) $\frac{4x^3y^2}{6x^7y}$

$= \boxed{\frac{2y}{3x^4}}$

f) $(-2)^4$

$= (-2)(-2)(-2)(-2)$

$= \boxed{16}$

g) -2^4

$= -1 \cdot 2^4$

$= -1 \cdot 16$

$= \boxed{-16}$

h) $5x^{-4}y^3 \cdot x^2y^{-1}$

$= 5x^{-2}y^2$

$= \boxed{\frac{5y^2}{x^2}}$

i) $\frac{1}{6^{-2}}$

$= 6^2$

$= \boxed{36}$

j) $9^{-3} \cdot 9^8$

$= \boxed{9^5}$

* NOT 81^5

k) $\frac{3x^2}{15x^{-3}y^{-4}}$

$= \boxed{\frac{x^5y^4}{5}}$

l) $\frac{y^{-5}}{y^{-4}}$

$= \frac{y^4}{y^5} = \boxed{\frac{1}{y}}$

m) $(y^{-5})^7$

$= y^{-35}$

$= \boxed{\frac{1}{y^{35}}}$

n) $(-2x)^3$

$= (-2x)(-2x)(-2x)$

$= \boxed{-8x^3}$

$= (-2)^3x^3$

$= \boxed{-8x^3}$

o) $(3x^5y^{-1})^{-2}$

$= 3^{-2}x^{-10}y^2$

$= \frac{y^2}{3^2x^{10}}$

$= \boxed{\frac{y^2}{9x^{10}}}$

p) $\left(\frac{y^2z^3}{5}\right)^{-3}$

$= \frac{y^{-6}z^{-9}}{5^{-3}}$

$= \frac{5^3}{y^6z^9}$

$= \boxed{\frac{125}{y^6z^9}}$

2.2 Rational Exponents

If n is a positive integer greater than 1 and $\sqrt[n]{a}$ is a real number then $a^{1/n} = \sqrt[n]{a}$.

★ **The denominator of the exponent tells you what type of root to take.**

Examples: Write an equivalent expression using radical notation and, if possible, simplify.

a) $25^{1/2}$

$= \sqrt[2]{25^1}$

$= \sqrt{25}$

$= \boxed{5}$

b) $64^{1/3}$

$= \sqrt[3]{64^1}$

$= \boxed{4}$

c) $(xy^2z)^{1/6}$

$= \boxed{\sqrt[6]{xy^2z}}$

d) $(36x^{10})^{1/2}$

$= \sqrt{36x^{10}}$

$= \boxed{6x^5}$

e) $2x^{1/4}$

$= \boxed{2\sqrt[4]{x}}$

f) $(2x)^{1/4}$

$= \boxed{\sqrt[4]{2x}}$

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[7]{2xy}$ $\boxed{(2xy)^{\frac{1}{7}}}$

b) $\sqrt[4]{\dfrac{ab^3}{7}}$ $\boxed{\left(\dfrac{ab^3}{7}\right)^{\frac{1}{4}}}$

c) $\sqrt{3z}$ $\boxed{(3z)^{\frac{1}{2}}}$

d) $3\sqrt{z}$ $\boxed{3z^{\frac{1}{2}}}$

e) $\sqrt[5]{xy^2z}$ $\boxed{(xy^2z)^{1/5}}$

Positive Rational Exponents

If m and n are positive integers (where $n \neq 1$) and $\sqrt[n]{a}$ exists, then $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$.

e.g.) $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$ or $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

Examples: Write an equivalent expression using radical notation and simplify.

a) $t^{5/6}$

$= \sqrt[6]{t^5}$

or $\left(\sqrt[6]{t}\right)^5$

b) $9^{3/2}$

$= \sqrt{9^3}$ or $\left(\sqrt{9}\right)^3$

$= 3^3 = \boxed{27}$

c) $64^{2/3}$

$= \sqrt[3]{64^2}$

or $\left(\sqrt[3]{64}\right)^2$

$= 4^2 = \boxed{16}$

d) $(2x)^{3/4}$

$= \boxed{\sqrt[4]{(2x)^3}}$

e) $2x^{3/4}$

$= \boxed{2\sqrt[4]{x^3}}$

Examples: Write an equivalent expression using exponential notation.

a) $\sqrt[3]{x^5}$ $\boxed{x^{5/3}}$

b) $\sqrt[7]{9^2}$ $\boxed{9^{2/7}}$

c) $\left(\sqrt[5]{6n}\right)^3$ $\boxed{(6n)^{3/5}}$

d) $6\sqrt[5]{n^3}$ $\boxed{6n^{3/5}}$

e) $\left(\sqrt[4]{2m}\right)^2$

$(2m)^{2/4}$

$= \boxed{(2m)^{1/2}}$

Negative Rational Exponents

For any rational number m/n, and any nonzero real number $a^{m/n}$, $a^{-m/n} = \frac{1}{a^{m/n}}$.

★ **The sign of the base is not affected by the sign of the exponent.**

Examples: Write an equivalent expression using positive exponents and, if possible, simplify.

a) $49^{-1/2}$

$= \frac{1}{49^{1/2}} = \frac{1}{\sqrt{49}} = \boxed{\frac{1}{7}}$

b) $(3mn)^{-2/5}$ $= \frac{1}{\sqrt[5]{(3mn)^2}}$

$= \boxed{\frac{1}{(3mn)^{2/5}}}$

c) $7x^{-2/3}$

$= \boxed{\frac{7}{x^{2/3}}}$

Laws of Exponents: The laws of exponents apply to rational exponents as well as integer exponents.

Examples: Use the laws of exponents to simplify.

a) $2^{2/5} \cdot 2^{1/5} = 2^{2/5+1/5}$

$= \boxed{2^{3/5}}$

b) $\frac{x^{7/3}}{x^{4/3}} = x^{7/3-4/3}$

$= x^{3/3} = \boxed{x}$

c) $\left(19^{2/5}\right)^{5/3}$

$= 19^{2/5 \cdot 5/3} = \boxed{19^{2/3}}$

d) $x^{1/2} \cdot x^{2/3}$

$= x^{1/2+2/3}$

$= \boxed{x^{7/6}}$

e) $y^{-4/7} \cdot y^{6/7}$

$= y^{-4/7+6/7}$

$= \boxed{y^{2/7}}$

f) $\frac{z^{3/4}}{z^{2/5}}$

$= z^{3/4-2/5}$

$= \boxed{z^{7/20}}$

g) $\frac{x^{3/4} \cdot x^{1/6} \cdot y}{y^{1/2}}$

$= x^{3/4+1/6} \cdot y^{1-1/2}$

$= \boxed{x^{11/12}\, y^{1/2}}$

h) $\frac{\left(2x^{2/5}y^{-1/3}\right)^5}{x^2 y}$

$= \frac{2^5 \cancel{x^2} y^{-5/3}}{\cancel{x^2} y}$

$= \frac{32}{y \cdot y^{5/3}} = \frac{32}{y^{1+5/3}}$

$= \boxed{\frac{32}{y^{8/3}}}$

To Simplify Radical Expressions using the Rules of Exponents:

1. Convert radical expressions to exponential expressions.
2. Use arithmetic and the laws of exponents to simplify.
3. Convert back to radical notation as needed.

Examples: Use rational exponents to simplify. Do not use exponents that are fractions in the final answer.

a) $\sqrt[8]{z^4}$

$z^{4/8}$

$= z^{1/2}$

$= \boxed{\sqrt{z}}$

b) $\left(\sqrt[3]{a^2bc^4}\right)^9$

$= (a^2bc^4)^{9/3}$

$= (a^2bc^4)^3 = \boxed{a^6b^3c^{12}}$

c) $\sqrt{x} \cdot \sqrt[4]{x}$

$= x^{1/2} \cdot x^{1/4}$

$= x^{3/4} = \boxed{\sqrt[4]{x^3}}$

d) $\sqrt[6]{y^2} \cdot \sqrt[9]{y}$

$y^{2/6} \cdot y^{1/9}$

$= y^{4/9} = \boxed{\sqrt[9]{y^4}}$

e) $\frac{\sqrt[3]{k}}{\sqrt[7]{k^2}}$

$= \frac{k^{1/3}}{k^{2/7}} = k^{1/3-2/7}$

$= k^{1/21}$

$= \boxed{\sqrt[21]{k}}$

f) $\frac{\sqrt[8]{m^4}}{\sqrt[6]{m}} = \frac{m^{4/8}}{m^{1/6}}$

$= m^{1/2-1/6}$

$= m^{2/6} = m^{1/3}$

$= \boxed{\sqrt[3]{m}}$

g) $\sqrt[4]{\sqrt[5]{x}}$

$= \left(x^{1/5}\right)^{1/4}$

$= x^{1/20}$

$= \boxed{\sqrt[20]{x}}$

h) $\sqrt[3]{2} \cdot \sqrt[5]{3}$

$= 2^{1/3} \cdot 3^{1/5}$

can't simplify

$\boxed{\sqrt[3]{2} \cdot \sqrt[5]{3}}$

2.3 Simplifying Radical Expressions

- ***Square Root:*** A number that you square (multiply by itself) to end up with a is called a square root of a. In symbols, $k = \sqrt{a}$ if $k^2 = a$.
- ***Radical Sign:*** The symbol $\sqrt{}$. The radical sign is used to indicate the *principal* (positive) square root of the number over which it appears.
- ***Radicand:*** The number under the radical sign.
- ***Perfect squares:*** Numbers that are the squares of rational numbers. Examples: 1, 4, 9, 81, $\frac{1}{36}$, $\frac{16}{25}$, etc.

Examples: Simplify each of the following:

a) $\sqrt{196} = \boxed{14}$

b) $\sqrt{625} = \boxed{25}$

c) $\sqrt{\frac{49}{81}} = \frac{\sqrt{49}}{\sqrt{81}} = \boxed{\frac{7}{9}}$

d) $\sqrt{y^4} = \boxed{y^2}$ $y^{4/2} = y^2$

e) $\sqrt{z^{14}} = \boxed{z^7}$ $z^{14/2} = z^7$

- ***nth Root:*** A number that you raise to the nth power (multiply by itself n times) to end up with a is called an nth root of a. In symbols, $k = \sqrt[n]{a}$ if $k^n = a$.
- ***Index:*** In the expression $\sqrt[n]{a}$, n is called the ***index.*** It tells you what root to take.

Examples: Simplify each expression, if possible.

a) $\sqrt[3]{125} = \boxed{5}$ because $5^3 = 125$

b) $\sqrt[4]{81} = \boxed{3}$ because $3^4 = 81$

c) $\sqrt[5]{32} = \boxed{2}$ because $2^5 = 32$

d) $\sqrt[3]{8x^6y^3} = 2x^2y$
$= \sqrt[3]{8} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{y^3}$
$= 2 \cdot x^{6/3} \cdot y^{3/3}$
$= \boxed{2x^2y^3}$

Simplified Radical Expressions:

- No perfect nth power factors in the radicand
- No exponents in the radicand bigger than the index
- No fractions in the radicand
- The index is as small as possible

To Simplify a Radical Expression with Index *n* by Factoring:

1. Write the radicand as the product of perfect nth powers and factors that are not perfect nth powers.
2. Rewrite the expression as the product of separate nth roots.
3. Simplify each expression containing the nth root of a perfect nth power.

To Simplify a Radical Expression with Index *n* Using a Factor Tree:

1. Make a factor tree. Split the radicand into its prime factors.
2. Circle groups of n identical factors.
3. List the number or variable from each group only ***once*** outside the radical.

4. Leave factors that are not part of a group under the radical.
5. Multiply the factors outside of the radical together. Do the same for the factors under the radical.

Examples: Simplify each expression.

a) $\sqrt{12}$ $\boxed{2\sqrt{3}}$

3 · 4
2 · 2

b) $\sqrt{40}$ $\boxed{2\sqrt{10}}$

4 · 10
2 2 2 5

c) $5\sqrt{72}$ $5\cdot2\cdot3\sqrt{2}$

12 · 6
4 3 2 3
2 2

$= \boxed{30\sqrt{2}}$

d) $\sqrt{20x^2y^3}$

$2 \cdot 10 \cdot x \cdot x \cdot y \cdot y \cdot y$
2 5

$= \boxed{2xy\sqrt{5y}}$

e) $2xy^2\sqrt{300x^3y^5}$

30 10
3 10 $\cdot x \cdot x \cdot x \cdot$
$y \cdot y \cdot y \cdot y \cdot y$

$2xy^2 \cdot 10 \cdot x \cdot y^2\sqrt{3xy}$

$= \boxed{20x^2y^4\sqrt{3xy}}$

f) $\sqrt[3]{54}$

9 · 6
3 3 3 2

$= \boxed{3\sqrt[3]{2}}$

g) $7\sqrt[3]{40}$

4 · 10
2 2 2 5

$7 \cdot 2\sqrt[3]{5}$

$= \boxed{14\sqrt[3]{5}}$

h) $\sqrt[3]{32t^7u^9}$

8 · 4 · $t \cdot t \cdot t \cdot t \cdot t \cdot t \cdot t$
2 4 2 2 $u \cdot u \cdot u \cdot u \cdot u \cdot u \cdot$
2 2 $u \cdot u \cdot u$

$= \boxed{2t^2u^3\sqrt[3]{4t}}$

i) $3m\sqrt[3]{40mn^6}$

4 · 10 $\cdot m \cdot n \cdot n \cdot n \cdot n \cdot$
2 2 2 5 $n \cdot n$

$3m \cdot 2n^2\sqrt[3]{5m}$

$= \boxed{6mn^2\sqrt[3]{5m}}$

j) $\sqrt[4]{240}$

24 · 10
12 2 2 5
2 6
2 3

$= \boxed{2\sqrt[4]{15}}$

k) $\sqrt[4]{x^6y^9z^3}$

$x \cdot x \cdot x \cdot x \cdot x \cdot x$
$y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$
$z \cdot z \cdot z$

$\boxed{xy^2\sqrt[4]{x^2yz^3}}$

$x^{6/4}$

l) $pr\sqrt[5]{p^7q^{23}r^{14}}$

$p \cdot p q^4 r \cdot r^2 \sqrt[5]{p^2q^3r^4}$

$= \boxed{p^2q^4r^3\sqrt[5]{p^2q^3r^4}}$

Operations with Radicals

Adding and Subtracting Radicals:

1. Simplify each radical completely.
2. Combine like radicals. When you add or subtract radicals, you can *only* combine radicals that have the same index and the same radicand. The radical itself (the root) does not change. You simply add or subtract the coefficients.

Like Radicals: Radicals with the same index *and* the same radicand.

Examples: Determine whether the following are like radicals. If they are not, explain why not.

a) $\sqrt{3}$ and $\sqrt{2}$ $\boxed{\text{No}}$
same index but different radicands

b) $4\sqrt{5}$ and $-3\sqrt{5}$ $\boxed{\text{yes}}$
same index & same radicand

c) $2\sqrt{x}$ and $\sqrt[3]{x}$ $\boxed{\text{No}}$
same radicand but different index

Examples: Add or subtract.

a) $5\sqrt{3x} - 7\sqrt{3x}$

$\boxed{12\sqrt{3x}}$

b) $4\sqrt{11} + 8\sqrt{11}$

$\boxed{12\sqrt{11}}$

c) $10\sqrt{6} + 3\sqrt{2} - 8\sqrt{6}$

$\boxed{2\sqrt{6} + 3\sqrt{2}}$

Simplify first...

d) $\sqrt{20} - \sqrt{50} + \sqrt{45}$

$2\sqrt{5} - 5\sqrt{2} + 3\sqrt{5}$

$= \boxed{5\sqrt{5} - 5\sqrt{2}}$

e) $2\sqrt{50} + 4\sqrt{500} - 6\sqrt{125}$

$10\sqrt{2} + 40\sqrt{5} - 30\sqrt{5}$

$= \boxed{10\sqrt{2} + 10\sqrt{5}}$

f) $\sqrt[3]{54} - 5\sqrt[3]{16} + \sqrt[3]{2}$

$3\sqrt[3]{2} - 10\sqrt[3]{2} + \sqrt[3]{2}$

$= \boxed{-6\sqrt[3]{2}}$

Don't make the following mistakes:

- $\sqrt{2} + \sqrt{5} \neq \sqrt{7}$
- $\sqrt{9+16} \neq 3+4$
- $\sqrt{m} - \sqrt{n} \neq \sqrt{m-n}$

Multiplying Radicals

The Product Rule for Radicals:

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$.

Caution: The product rule doesn't work if you are trying to multiply the even roots of negative numbers, because those roots are not real numbers. For example, $\sqrt{-2} \cdot \sqrt{-8} \neq \sqrt{16}$.

Re-write the radical in terms of i first, and then multiply.
For example, $\sqrt{-2} \cdot \sqrt{-8} = i\sqrt{2} \cdot i\sqrt{8} = i^2\sqrt{16} = (-1) \cdot \sqrt{16} = -4$

Caution: The product only applies when the radicals have the same index: $\sqrt[3]{5} \cdot \sqrt[4]{6} \neq \sqrt[12]{30}$.

Examples: Multiply.

a) $\sqrt{7} \cdot \sqrt{5}$ $= \boxed{\sqrt{35}}$

b) $5\sqrt{2} \cdot \sqrt{8}$

$= 5\sqrt{16}$

$= 5 \cdot 4 = \boxed{20}$

c) $2\sqrt{5} \cdot 7\sqrt{15}$

$= 14\sqrt{75}$

$= 14 \cdot 5\sqrt{3} = \boxed{70\sqrt{3}}$

d) $\sqrt{3} \cdot \sqrt{3}$ $= \sqrt{9} = \boxed{3}$

e) $\left(\sqrt{8}\right)^2$ $= \boxed{8}$

f) $\left(3\sqrt{11}\right)^2$

$= 3\sqrt{11} \cdot 3\sqrt{11}$

$= 9 \cdot 11 = \boxed{99}$

g) $\sqrt[3]{3} \cdot \sqrt[3]{9}$ $= \sqrt[3]{27}$

$= \boxed{3}$

h) $2\sqrt[3]{10} \cdot 6\sqrt[3]{25}$

$= 12\sqrt[3]{250}$

2.4 Radical Expressions, Multiply and Divide (Rationalizing the Denominator)

Question: Can you add and subtract radicals the same way you multiply and divide them?
e.g.) since $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$, does $\sqrt{a}+\sqrt{b}=\sqrt{a+b}$? **NO!!!!!!!!!!**

Don't make the following mistakes:

- $\sqrt{x^2-4}\neq x-2$
- $\left(\sqrt{x}+\sqrt{y}\right)^2\neq x+y$

Multiplying Radical Expressions: Use the Product Property. Use the Distributive Property and FOIL to multiply radical expressions with more than one term.

Examples: Multiply.

a) $\sqrt{3}\left(5+\sqrt{30}\right)$

$5\sqrt{3}+\sqrt{90}$

$9 \cdot 10$

$3\ 3$

$=\boxed{5\sqrt{3}+3\sqrt{10}}$

b) $\sqrt{2}\left(\sqrt{6}-3\sqrt{2}\right)$

$\sqrt{12}+3\sqrt{4}$

$4 \cdot 3 \quad 2\ 2$

$2\ 2$

$=\boxed{2\sqrt{3}+6}$

c) $\left(\sqrt{5}-\sqrt{6}\right)\left(\sqrt{7}+1\right)$

$\boxed{\sqrt{35}+\sqrt{5}-\sqrt{42}-\sqrt{6}}$

d) $\left(5\sqrt{3}-4\sqrt{2}\right)\left(\sqrt{3}+\sqrt{2}\right)$

$5\sqrt{9}+5\sqrt{6}-4\sqrt{6}-4\sqrt{4}$

$=15+\sqrt{6}-8$

$=\boxed{7+\sqrt{6}}$

e) $\left(4\sqrt{3}-1\right)^2$

$(4\sqrt{3}-1)(4\sqrt{3}-1)$

$=16\sqrt{9}-4\sqrt{3}-4\sqrt{3}+1$

$=16\cdot 3-8\sqrt{3}+1$

$=48-8\sqrt{3}+1$

$=\boxed{49-8\sqrt{3}}$

f) $\left(\sqrt{2}+5\right)\left(\sqrt{2}-5\right)$

$\sqrt{4}-\cancel{5\sqrt{2}}+\cancel{5\sqrt{2}}-25$

$=2-25$

$=\boxed{-23}$

Dividing Radicals

The Quotient Rule for Radicals

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$, where $b\neq 0$, $\sqrt[n]{\dfrac{a}{b}}=\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$.

Examples: Simplify.

a) $\sqrt{\dfrac{9}{25}}=\dfrac{\sqrt{9}}{\sqrt{25}}=\boxed{\dfrac{3}{5}}$

b) $\sqrt[3]{\dfrac{x^3}{27}}=\dfrac{\sqrt[3]{x^3}}{\sqrt[3]{27}}$

$=\boxed{\dfrac{x}{3}}$

c) $\sqrt{\dfrac{m^3}{16n^2}}=\dfrac{\sqrt{m^3}}{\sqrt{16n^2}}$

$=\boxed{\dfrac{\sqrt{m^3}}{4n}}$

d) $\sqrt[3]{\dfrac{250y^{11}}{8x^6}}=\dfrac{\sqrt[3]{250y^{11}}}{\sqrt[3]{8x^6}}$

$25\cdot 10$

$5\ 5\ 5\ 2$

$=\boxed{\dfrac{5y^3\sqrt[3]{2y^2}}{2x^2}}$

Examples: Divide and, if possible, simplify.

a) $\dfrac{\sqrt{72}}{\sqrt{2}} = \sqrt{36} = \boxed{6}$

b) $\dfrac{\sqrt{50x}}{2\sqrt{2}} = \dfrac{\sqrt{25x}}{2} = \boxed{\dfrac{5\sqrt{x}}{2}}$

c) $\dfrac{7\sqrt[3]{48x^4y^8}}{\sqrt[3]{6y^2}} = 7\sqrt[3]{8x^4y^6}$

$= 7 \cdot 2xy^2\sqrt[3]{x}$

$= \boxed{14xy^2\sqrt[3]{x}}$

Rationalizing Denominators with One Term:

Rationalizing the denominator means to write the expression as an equivalent expression but without a radical in the denominator. To do this, multiply by 1 under the radical or multiply by 1 outside the radical to make the denominator a perfect power.

Examples: Rationalize each denominator.

a) $\sqrt{\dfrac{2}{3}} = \dfrac{\sqrt{2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{\sqrt{6}}{\sqrt{9}}$

$= \boxed{\dfrac{\sqrt{6}}{3}}$

b) $\dfrac{5}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{5\sqrt{3}}{\sqrt{9}}$

$= \boxed{\dfrac{5\sqrt{3}}{3}}$

c) $\dfrac{5}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{5\sqrt{5}}{2\sqrt{25}}$

$= \dfrac{5\sqrt{5}}{2 \cdot 5}$

$= \dfrac{5\sqrt{5}}{10} = \boxed{\dfrac{\sqrt{5}}{2}}$

d) $\dfrac{3-\sqrt{5}}{\sqrt{11}} \cdot \dfrac{\sqrt{11}}{\sqrt{11}}$

$= \dfrac{3\sqrt{11} - \sqrt{55}}{\sqrt{121}}$

$= \boxed{\dfrac{3\sqrt{11} - \sqrt{55}}{11}}$

Rationalizing Denominators with Two Terms:

To do this, multiply by 1 under the radical or multiply by 1 outside the radical to make the denominator a perfect power. However, since the denominator now has two terms, we will have to multiply by the **conjugate** of the denominator.

Conjugate of a binomial Radical Expression: Conjugates have the same first term, with the second terms being opposites. For example, these two expressions are conjugates: $3-\sqrt{2}$ and $3+\sqrt{2}$.

What happens when you multiply these conjugates together?

$(3-\sqrt{2})(3+\sqrt{2}) = 9 + \cancel{3\sqrt{2}} - \cancel{3\sqrt{2}} - \sqrt{4}$

$= 9 - 2$

$= 7$

Examples: Find the conjugate of each number.

a) $4+\sqrt{5}$

$\boxed{4-\sqrt{5}}$

b) $-3-\sqrt{7}$

$\boxed{-3+\sqrt{7}}$

c) $\sqrt{15}$

$\boxed{-\sqrt{15}}$

Examples: Rationalize each denominator by multiplying by the conjugate.

a) $\dfrac{4}{(2+\sqrt{2})} \cdot \dfrac{(2-\sqrt{2})}{(2-\sqrt{2})}$

$= \dfrac{8-4\sqrt{2}}{4-\cancel{2\sqrt{2}}+\cancel{2\sqrt{2}}-\sqrt{4}}$

$= \dfrac{8-4\sqrt{2}}{4-2}$

$= \dfrac{8-4\sqrt{2}}{2}$

Can we simplify further?

YES! $= \boxed{\dfrac{4-2\sqrt{2}}{2}}$

b) $\dfrac{5}{(8-\sqrt{3})} \cdot \dfrac{(8+\sqrt{3})}{(8+\sqrt{3})}$

$= \dfrac{40+5\sqrt{3}}{64+\cancel{8\sqrt{3}}-\cancel{8\sqrt{3}}-\sqrt{9}}$

$= \dfrac{40+5\sqrt{3}}{64-3}$

$= \boxed{\dfrac{40+5\sqrt{3}}{61}}$

c) $\dfrac{(5-\sqrt{3})}{(2+\sqrt{5})} \cdot \dfrac{(2-\sqrt{5})}{(2-\sqrt{5})}$

$= \dfrac{10-5\sqrt{5}-2\sqrt{3}+\sqrt{15}}{4-\cancel{2\sqrt{5}}+\cancel{2\sqrt{5}}-\sqrt{25}}$

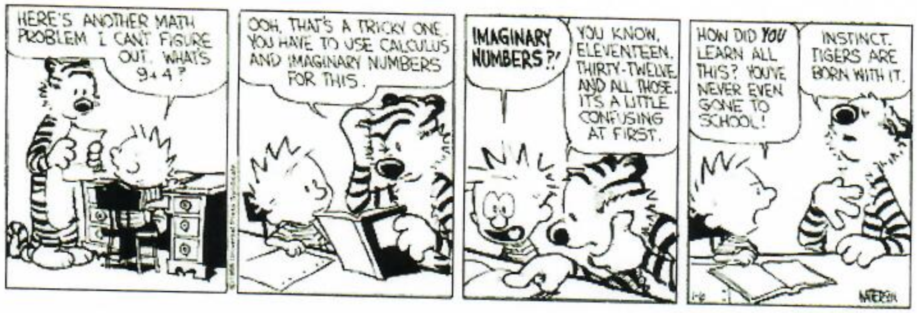
$= \dfrac{10-5\sqrt{5}-2\sqrt{3}+\sqrt{15}}{4-5}$

$= \dfrac{10-5\sqrt{5}-2\sqrt{3}+\sqrt{15}}{-1}$

$= \boxed{-10+5\sqrt{5}+2\sqrt{3}-\sqrt{15}}$

2.5 Simplifying with Complex Numbers

Imaginary Numbers



For centuries, mathematicians kept running into problems that required them to take the square roots of negative numbers in the process of finding a solution. None of the numbers that mathematicians were used to dealing with (the "real" numbers) could be multiplied by themselves to give a negative. These square roots of negative numbers were a new type of number. The French mathematician René Descartes named these numbers "imaginary" numbers in 1637. Unfortunately, the name "imaginary" makes it sound like imaginary numbers don't exist. They do exist, but they seem strange to us because most of us don't use them in day-to-day life, so we have a hard time visualizing what they mean. However, imaginary numbers are extremely useful (especially in electrical engineering) and make many of the technologies we use today (radio, electrical circuits) possible.

The number i: i is the number whose square is -1. That is, $\boxed{i=\sqrt{-1} \text{ and } i^2=-1.}$

We define the square root of a negative number as follows:

$$\sqrt{-x}=\sqrt{-1\cdot x}=\sqrt{-1}\cdot\sqrt{x}=i\sqrt{x} \text{ or } \sqrt{x}\cdot i.$$

Examples: Express in terms of i.

a) $\sqrt{-64}$ $= i\sqrt{64}$

$= \boxed{8i}$

b) $\sqrt{-12}$ $= i\sqrt{12}$

2 6

2 3

$= \boxed{2i\sqrt{3}}$

c) $-\sqrt{-49}$

$-i\sqrt{49}$

$= \boxed{-7i}$

d) $-\sqrt{-18}$

$-i\sqrt{18}$

9 2

3 3

$\boxed{-3i\sqrt{2}}$

Imaginary Number: A number that can be written in the form $a+bi$, where a and b are real numbers and $b \neq 0$. **Any number with an i in it is imaginary.**

Complex Number: A number that can be written in the form $a+bi$, where a and b are real numbers. (a or b or both can be 0.) **The set of complex numbers is the set containing all of the real numbers and all of the imaginary numbers.**

Adding or Subtracting Complex Numbers

★ i acts like any other variable in addition and subtraction problems. Distribute any negative signs and combine like terms (add or subtract the real parts and add or subtract the imaginary parts). Write your answer with the real part first, then the imaginary part.

Examples: Add or subtract and simplify.

a) $(2+5i)+(1-3i)$

$\boxed{3+2i}$

b) $(4-3i)-(-2+5i)$

$4-3i+2-5i$

$\boxed{6-8i}$

c) $(-3-7i)-(-6)$

$-3-7i+6$

$\boxed{3-7i}$

d) $5i-(1-i)$

$5i-1+i$

$\boxed{-1+6i}$

Multiplying Complex Numbers

Multiplying Complex Numbers:

- To multiply imaginary numbers, first write any square roots of negative numbers in terms of i.
- Multiply as usual by distributing, FOILing, and using exponent rules. Treat i like any other variable.
- Use the fact that $i^2=-1$. Anywhere you see an i^2, change it to a -1.
  - $8i^2=8(-1)=-8$
  - $-3i^2=(-3)(-1)=3$

Examples: Multiply and simplify. If the answer is imaginary, write it in the form $a+bi$.

* deal with negatives under the square root First!

a) $\sqrt{-9}\cdot\sqrt{-4}$

$=i\sqrt{9}\cdot i\sqrt{4}$

$=3i\cdot 2i=6i^2$

$=\boxed{-6}$

b) $\sqrt{-3}\cdot\sqrt{-5}$

$i\sqrt{3}\cdot i\sqrt{5}$

$=i^2\sqrt{15}$

$=\boxed{-\sqrt{15}}$

c) $-2i\cdot 7i$

$-14i^2$

$=-14(-1)$

$=\boxed{14}$

d) $-3i\cdot i\sqrt{5}$

$=-3i^2\sqrt{5}$

$=-3(-1)\sqrt{5}$

$=\boxed{3\sqrt{5}}$

e) $3i(2-i)$

$6i-3i^2$

$=6i-3(-1)$

$=6i+3=\boxed{3+6i}$

f) $(7+3i)(9-8i)$

$63-56i+27i-24i^2$

$=63-29i+24$

$=\boxed{87-29i}$

g) $(2-i)^2$

$(2-i)(2-i)$

$=4-2i-2i+i^2$

$=4-4i-1$

$=\boxed{3-4i}$

h) $(3-4i)(3+4i)$

$9+\cancel{12i}-\cancel{12i}-16i^2$

$=9+16=\boxed{25}$

Simplify a Power of i: Express the given power of i in terms of powers of i^2, and use the fact that $i^2=-1$.

Examples: Simplify each expression.

a) $i^{22}=(i^2)^{11}$

$=(-1)^{11}=\boxed{-1}$

b) i^{33}

$=(i^2)^{16}\cdot i$

$=(-1)^{16}\cdot i$

$=1\cdot i=\boxed{i}$

c) $i^{72}=(i^2)^{36}$

$=(-1)^{36}$

$=\boxed{1}$

d) $i^{47}=(i^2)^{23}\cdot i$

$=(-1)^{23}\cdot i$

$=-1\cdot i$

$=\boxed{-i}$

2.6 Dividing Complex Numbers

Conjugate of a Complex Number: The **complex conjugate** of a complex number $a+bi$ is $a-bi$.
$(a+bi)(a-bi)=a^2+b^2$.

Examples: Find the conjugate of each number.

a) $-2+4i$ $\boxed{-2-4i}$

b) $1-i$ $\boxed{1+i}$

c) $-3i$ $\boxed{3i}$

Dividing Complex Numbers: Multiply both the numerator and the denominator by the complex conjugate of the denominator.

Examples: Divide and simplify to the form $a+bi$.

a) $\frac{7}{3i}\cdot\frac{-3i}{-3i}$

$=\frac{-21i}{-9i^2}=\frac{-21i}{-9(-1)}$

$=\boxed{\frac{-21i}{9}}$

b) $\frac{(2+6i)}{-5i}\cdot\frac{5i}{5i}$

$=\frac{10i+30i^2}{-25i^2}$

$=\frac{10i+30(-1)}{-25(-1)}$

$=\frac{10i+30}{25}$

$=\frac{30+10i}{25}$

$=\boxed{\frac{6+2i}{5}}$

c) $\frac{9i}{-7+6i}\frac{(-7-6i)}{(-7-6i)}$

$=\frac{-63i-54i^2}{49+\cancel{42i}-\cancel{42i}-36i^2}$

$=\frac{-63i+54}{49+36}$

$=\boxed{\frac{54-63i}{85}}$

d) $\frac{2+3i}{4-5i}\frac{(4+5i)}{(4+5i)}$

$=\frac{8+10i+12i+15i^2}{16+\cancel{20i}-\cancel{20i}-25i^2}$

$=\frac{8+22i-15}{16+25}$

$=\boxed{\frac{-7+22i}{41}}$