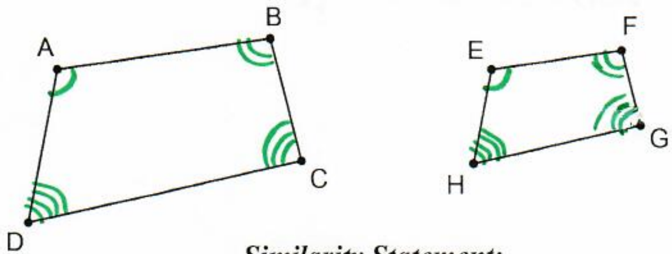
SM2H 7.5 Similarity Notes

Congruent Figures: Same shape and same size.
Similar Figures: Same shape.

If two polygons are similar, then:

- Their **corresponding angles are congruent.**
- The lengths of their **corresponding sides are proportional.**

Examples:



Similarity Statement:
ABCD ~ EFGH

1. List all pairs of congruent angles.

$\angle A \cong \angle E$, $\angle B \cong \angle F$
$\angle C \cong \angle G$, $\angle D \cong \angle H$

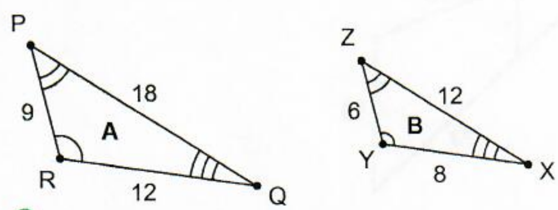
2. Write a ***statement of proportionality*** for the sides.

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

Scale Factor: The ratio of the lengths of two corresponding sides in similar polygons.

Examples: Decide whether each set of figures are similar. If they are similar, write a similarity statement and find the scale factor.

1.

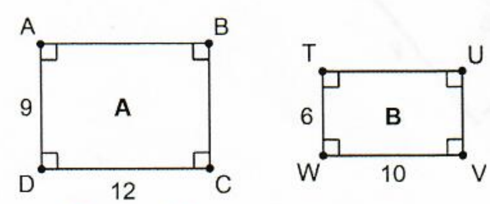


$$\frac{9}{6} = \frac{18}{12} = \frac{12}{8}$$
1.5 1.5 1.5

$\triangle PQR \sim \triangle ZXY$

Scale factor
1.5 or 2/3

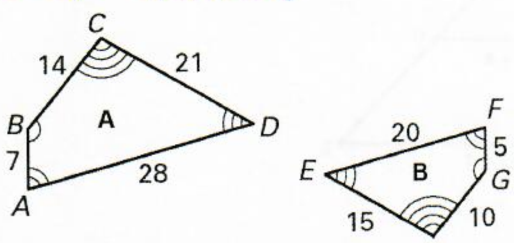
2.



$$\frac{9}{6} \neq \frac{12}{10}$$
1.5 1.2

not similar

3.



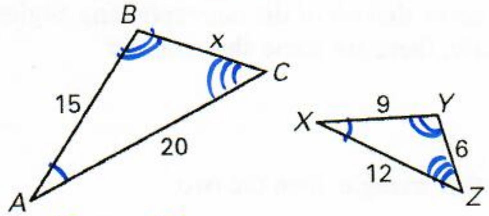
$$\frac{7}{5} = \frac{28}{20} = \frac{21}{15} = \frac{14}{10}$$
1.4 1.4 1.4 1.4

Scale factor
1.4 or $\frac{5}{7}$

BADC ~ GFEH

Examples: $\Delta ABC \sim \Delta XYZ$. Find the value of x.

1.

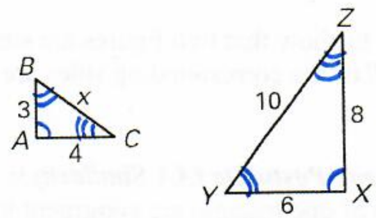


$\frac{15}{x} = \frac{9}{6}$

$9x = 90$

$\boxed{x = 10}$

2.

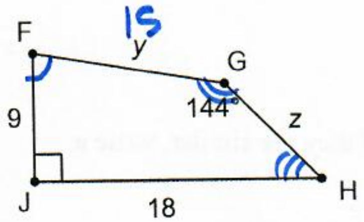


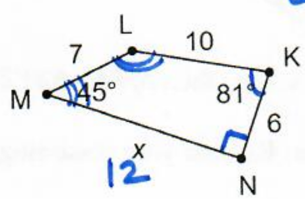
$\frac{3}{x} = \frac{6}{10}$

$30 = 6x$

$\boxed{5 = x}$

Examples: In the diagram below, $FGHJ \sim KLMN$.





1. List all pairs of congruent angles.

$\angle F \cong \angle K$, $\angle G \cong \angle L$, $\angle H \cong \angle M$, $\angle J \cong \angle N$

2. Write a statement of proportionality.

$\frac{FG}{KL} = \frac{GH}{LM} = \frac{HJ}{MN} = \frac{JF}{NK}$

3. Find $m\angle F$.

$81°$

4. Find $m\angle H$.

$45°$

5. Find $m\angle L$.

$144°$

6. Find $m\angle N$.

$90°$

7. Find the value of x.

$\frac{x}{18} = \frac{6}{9}$ $\quad 9x = 108$ $\quad \boxed{x = 12}$

8. Find the value of y.

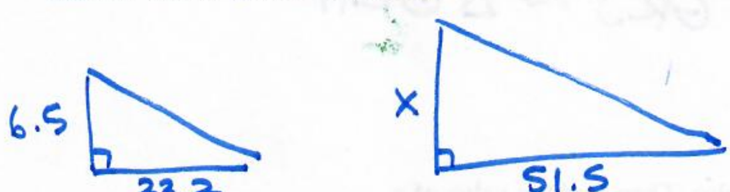
$\frac{y}{10} = \frac{9}{6}$ $\quad 6y = 90$ $\quad \boxed{y = 15}$

9. Find the value of z.

$\frac{z}{7} = \frac{9}{6}$ $\quad 6z = 63$ $\quad \boxed{z = 10.5}$

Examples:

1. A 6.5 ft. tall car standing next to an adult elephant casts a 33.2 ft. shadow. If the adult elephant casts a shadow that is 51.5 ft. long, then how tall is the elephant?

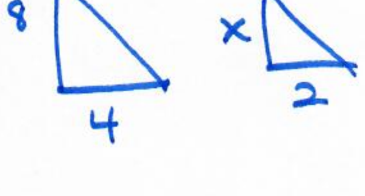


$\frac{6.5}{x} = \frac{33.2}{51.5}$

$33.2x = 334.75$

$\boxed{x = 10.08 \text{ feet}}$

2. A telephone booth that is 8 ft. tall casts a shadow that is 4 ft. long. Find the height of a nearby lawn ornament that casts a 2 ft. shadow.



$\frac{8}{x} = \frac{4}{2}$

$16 = 4x$

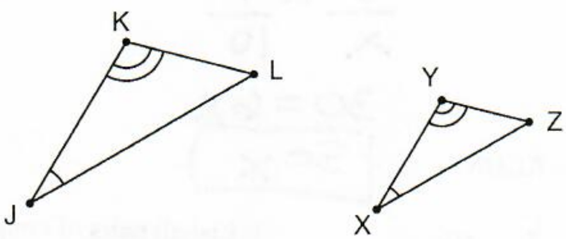
$\boxed{4 = x \text{ feet}}$

Triangle Similarity Theorems

So far, if we wanted to show that two figures are similar, we've had to show that *all* of the corresponding angles are congruent and *all* of the corresponding sides are proportional. Luckily, there are some shortcuts for triangles.

Angle-Angle Similarity Postulate (AA Similarity):

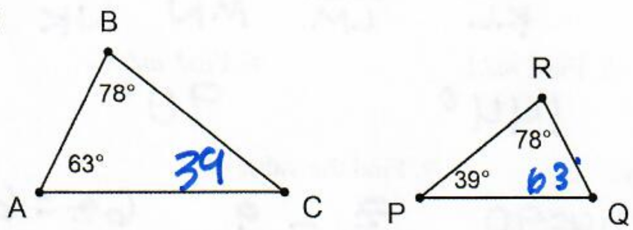
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



If $\angle J \cong \angle X$ and $\angle K \cong \angle Y$, then $\Delta JKL \sim \Delta XYZ$.

Examples: Determine whether the triangles are similar. Explain your reasoning. If they are similar, write a similarity statement.

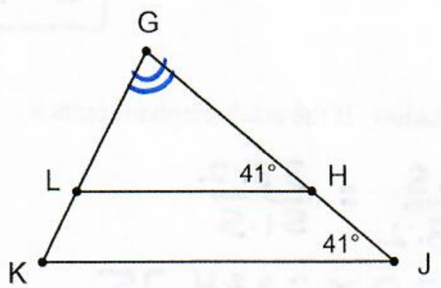
a)



yes, AA

$\Delta ABC \sim \Delta QRP$

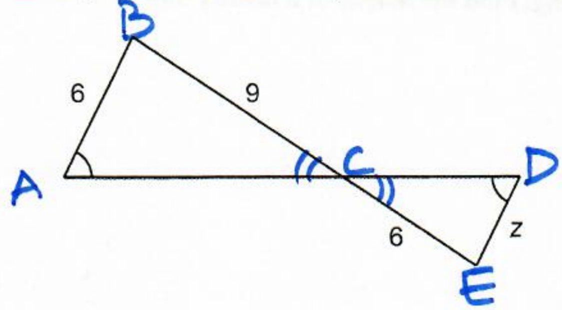
b)



yes, AA

$\Delta GKJ \sim \Delta GLH$

Example: Write a similarity statement for the triangles. Then find the value of *z*.



$\Delta ABC \sim \Delta DEC$

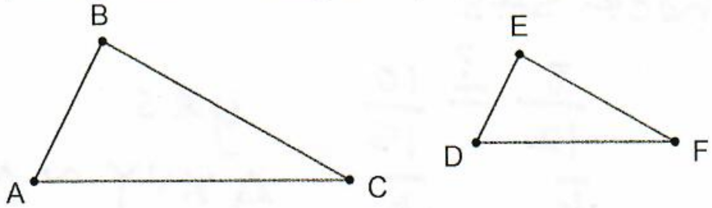
$\frac{6}{z} = \frac{9}{6}$

$36 = 9z$

$\boxed{4 = z}$

Side-Side-Side Similarity Theorem (SSS Similarity)

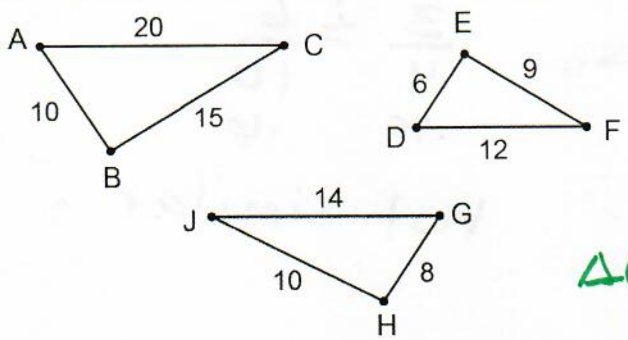
If the corresponding sides of two triangles are proportional, then the triangles are similar.



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then $\Delta ABC \sim \Delta DEF$.

★ **TIP:** When testing for SSS similarity, compare the shortest sides, longest sides, and medium sides.

Example: Is either ΔDEF or ΔGHJ similar to ΔABC?



ΔDEF

$\frac{10}{6} = \frac{15}{9} = \frac{20}{12}$ yes

$1.\overline{6}$ $1.\overline{6}$ $1.\overline{6}$

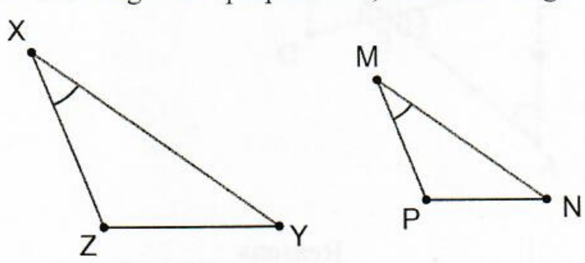
ΔGHJ

$\frac{10}{8} = \frac{15}{10} = \frac{20}{14}$ no

1.25 1.5 1.43

Side-Angle-Side Similarity Theorem (SAS Similarity)

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides that include these angles are proportional, then the triangles are similar.

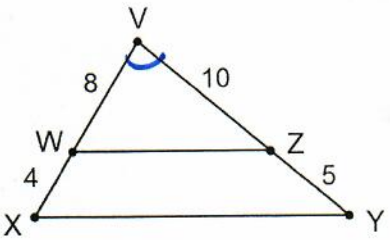


If $\angle X \cong \angle M$ and $\frac{PM}{ZX} = \frac{MN}{XY}$, then $\Delta XYZ \sim \Delta MNP$.

Examples: Determine whether the triangles are similar. If they are similar, write a similarity statement and determine the scale factor.

check SAS

a)



$$\frac{8}{12} \stackrel{?}{=} \frac{10}{15}$$

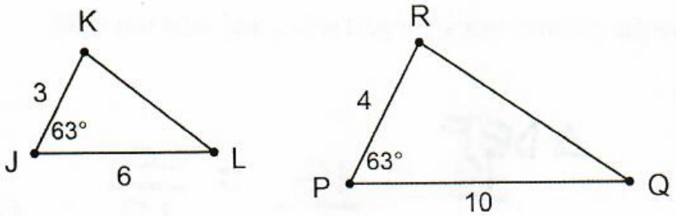
$.\overline{6}$ $\quad$ $.\overline{6}$

yes

$\triangle XVY \sim \triangle WVZ$

scale factor $\frac{2}{3}$ or 1.5

b)



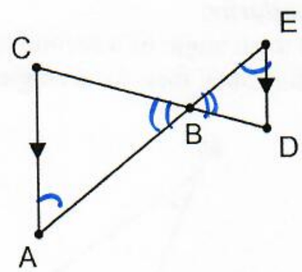
$$\frac{3}{4} \stackrel{?}{=} \frac{6}{10}$$

.75 $\quad$ .6

not similar

Complete the following proof:

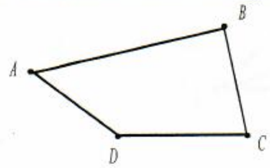
Given: $\overline{AC} \parallel \overline{DE}$
Prove: $\Delta ABC \sim \Delta EBD$



Statements	Reasons
1. $\overline{AC} \parallel \overline{DE}$	1. Given
2. $\angle A \cong \angle E$	2. alternate interior angles are congruent
3. $\angle CBA \cong \angle DBE$	3. Vertical Angles Theorem
4. $\Delta ABC \sim \Delta EBD$	4. AA Similarity Theorem

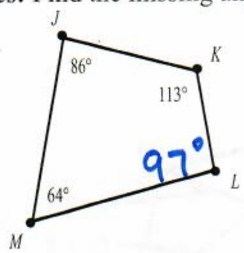
SM2H 7.6 Quadrilaterals Notes

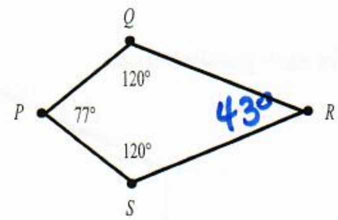
Quadrilateral Interior Angles Theorem: The measures of the interior angles of a quadrilateral add up to 360°.



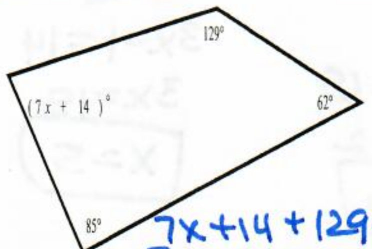
$$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$$

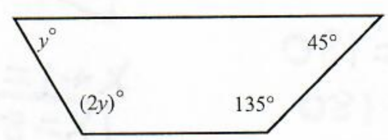
Examples: Find the missing angle measures in each quadrilateral.





Examples: Find the value of the variable in each quadrilateral.





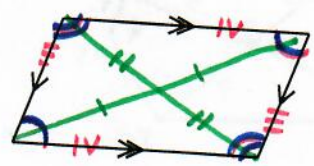
$7x+14+129+62+85=360$
$7x+290=360$
$7x=70$
$\boxed{x=10}$

$y+2y+135+45=360$
$3y+90=360$
$3y=270$
$\boxed{y=90}$

Parallelogram: A quadrilateral with two pairs of parallel sides.

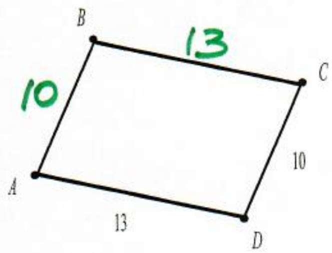
Properties:

- Opposite sides are parallel. (Definition)
- Opposite sides are congruent. (Theorem)
- Opposite angles are congruent. (Theorem)
- Consecutive angles are supplementary. (Theorem)
- Diagonals bisect each other, (Theorem)



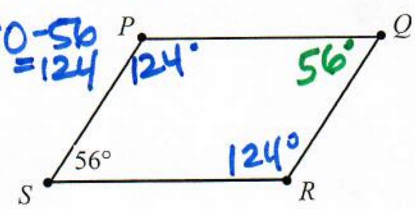
Examples:

Find AB and BC in $ABCD$.



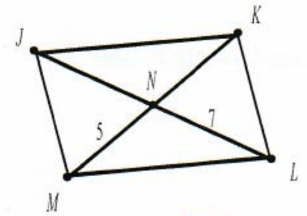
Find the missing angle measures in $PQRS$.

$180-56$
$=124$



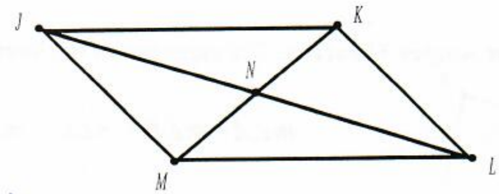
JKLM is a parallelogram. Find the requested measures.

Find *JN* and *KN*.



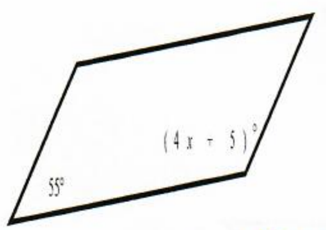
JN=7 (diagonals bisect each other)
KN=5

Find *JN* and *NL* if *JL* = 20.

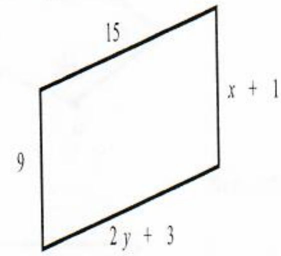


JN = 10
NL = 10

Find the value of the variables in each parallelogram.

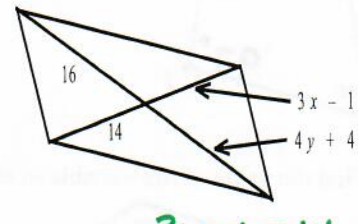


$55 + 4x + 5 = 180$
$4x + 60 = 180$
$4x = 120$
$\boxed{x = 30}$



$x + 1 = 9$
$\boxed{x = 8}$

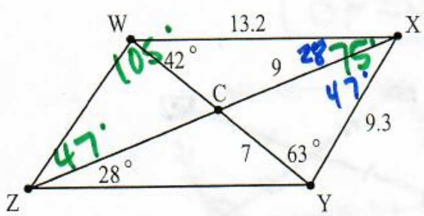
$2y + 3 = 15$
$2y = 12$
$\boxed{y = 6}$



$3x - 1 = 14$
$3x = 15$
$\boxed{x = 5}$

$4y + 4 = 16$
$4y = 12$
$\boxed{y = 3}$

In the diagram below, *WXYZ* is a parallelogram. Find the requested measures.



$m\angle XWZ =$ 105°

$m\angle WXY =$ 75°

$m\angle WZC =$ 47°

$m\angle WCX =$ 110°
180 - 42 - 28

$YZ =$ 13.2

$CZ =$ 9

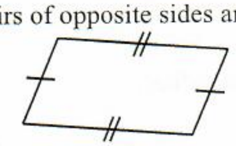
$WY =$ 14

$WZ =$ 9.3

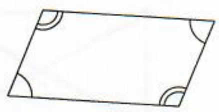
To Prove that a Quadrilateral is a Parallelogram:

Show that *both* pairs of opposite sides are parallel. (Definition of parallelogram)

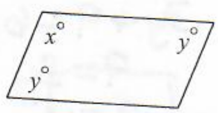
Show that *both* pairs of opposite sides are congruent. (Theorem)



Show that *both* pairs of opposite angles are congruent. (Theorem)

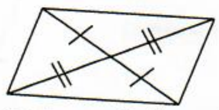


Show that one angle is supplementary to both of its consecutive angles. (Theorem)

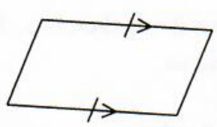


$x° + y° = 180°$

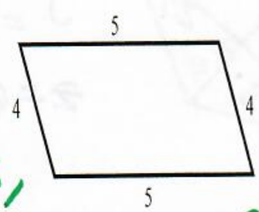
Show that the diagonals bisect each other. (Theorem)



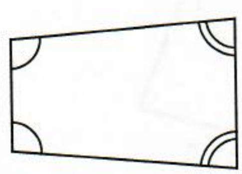
Show that one pair of sides are *both* parallel and congruent. (Theorem)



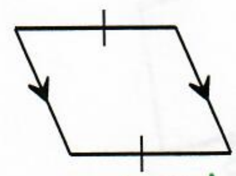
Examples: Decide whether each quadrilateral is a parallelogram. Explain your reasoning. Hint: On each problem, list everything that the diagram tells you. Then think about whether you can use that information to say anything else about the diagram. Finally, decide whether you have enough information to use <u>one</u> of the theorems above.



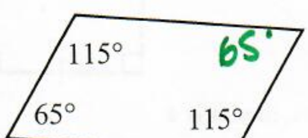
yes, both pairs of opposite sides are congruent



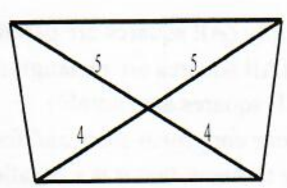
not a parallelogram



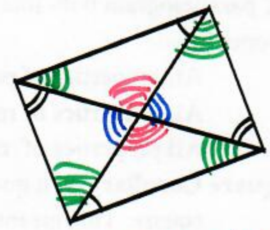
not a parallelogram



yes, both pairs of opposite angles are congruent

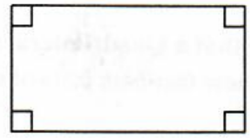


not a parallelogram



yes, both pairs of opposite angles are congruent.

Rectangle: A parallelogram with four right angles.



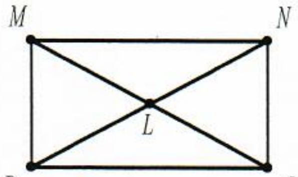
Properties:

- All properties of parallelograms apply (Rectangles are parallelograms)
- Four right angles (Definition)
- Congruent diagonals (Theorem)

Rectangle Corollary: If a quadrilateral has four right angles, then it is a rectangle. This means you don't have to know that it is a parallelogram to show it is a rectangle.

Examples: Each of the quadrilaterals below is a rectangle. Find the requested values.

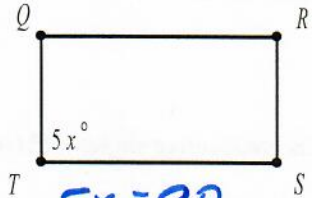
If $MO = 10$, find NP, ML, and NL.



NP = 10

ML = 5 NL = 5

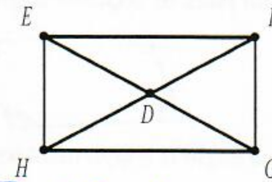
Find the value of x.



$5x = 90$

$x = 18$

Find the value of y, EG and DG.



$EG = 3y + 9$

$FH = 6y$

$3y + 9 = 6y$

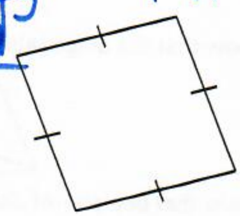
$-9 = 3y$

$\boxed{3 = y}$

$EG = 3(3) + 9 = 18$

$FH = 6(3) = 18$

Rhombus: A parallelogram with four congruent sides.



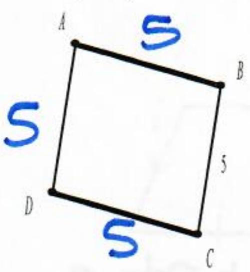
Properties:

- All properties of parallelograms apply (Rhombi are parallelograms)
- All four sides are congruent (Definition)
- Diagonals are perpendicular bisectors (Theorem)
- Each diagonal bisects a pair of opposite angles

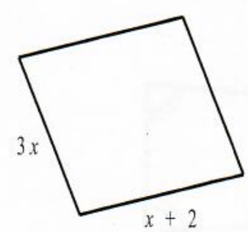
Rhombus Corollary: If a quadrilateral has four congruent sides, then it is a rhombus. This means you don't have to know that it is a parallelogram to show it is a rhombus.

Examples: Each of the quadrilaterals below is a rhombus. Find the requested values.

Find AB, CD, and AD.



Find the value of x.

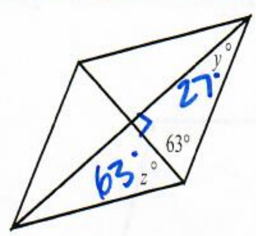


$3x = x + 2$

$2x = 2$

$x = 1$

Find the values of y and z.

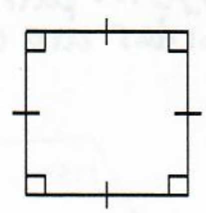


$y = 180 - 90 - 63$

$= 27°$

$z = 63°$

Square: A parallelogram with four congruent sides ***and*** four right angles.



Properties:

- All properties of parallelograms apply (All squares are parallelograms)
- All properties of rectangles apply (All squares are rectangles)
- All properties of rhombi apply (All squares are rhombi)

Square Corollary: If a quadrilateral has four congruent sides and four right angles, then it is a square. This means you don't have to know that it is a parallelogram to show it is a square.

Trapezoid: A quadrilateral with *exactly one* pair of parallel sides.

Bases of a trapezoid: The parallel sides of a trapezoid.

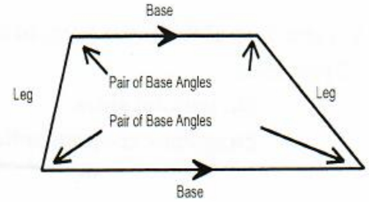
Base angles: Two angles that share a base.

A trapezoid has two pairs of base angles.

Legs of a trapezoid: The non-parallel sides of a trapezoid.

Properties:

The angles on either side of each leg are supplementary (Same-side interior angles).



Midsegment of a trapezoid: The segment that joins the midpoints of the legs.

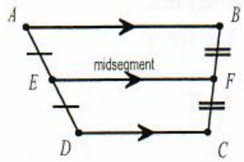
Properties of the midsegment of a trapezoid:

Bisects the legs (definition)

Parallel to the two bases

Length is the average of the lengths of the bases (add the lengths of the bases and divide by two).

$$EF = \frac{AB + CD}{2}$$

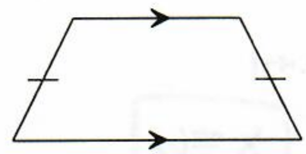


Isosceles Trapezoid: A trapezoid with congruent legs.

Properties:

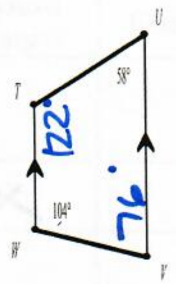
Diagonals are congruent.

Base angles are congruent.

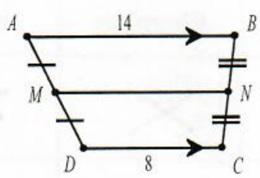


Examples:

Find the missing angle measures.



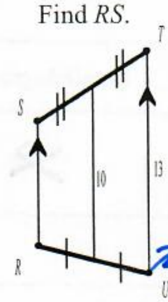
Find the length of midsegment $\overline{MN}$.



$\frac{14+8}{2} = \frac{22}{2}$

$\boxed{11}$

Find RS.



$2 \cdot \frac{x+13}{2} = 10 \cdot 2$

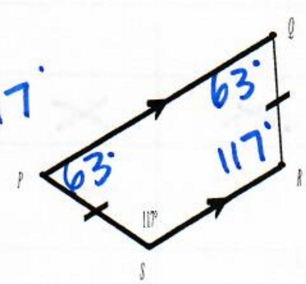
$x+13=20$

$x=7$

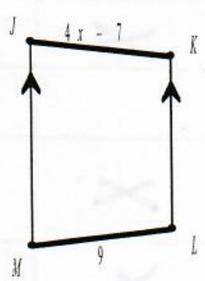
$\boxed{RS=7}$

PQRS is an isosceles trapezoid.
Find the missing angle measures.

180-117°



JKLM is an isosceles trapezoid.
Find the value of x.

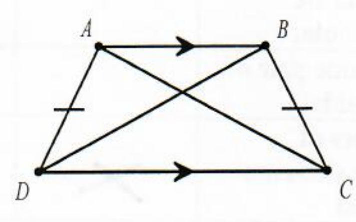


$4x-7=9$
$+7 \;\; +7$

$4x=16$

$\boxed{x=4}$

ABCD is an isosceles trapezoid.
Find AC if $BD = 10$.



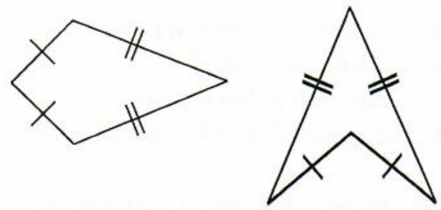
$AC=10$

Kite: A quadrilateral with two pairs of congruent consecutive sides and no congruent opposite sides.

Properties:

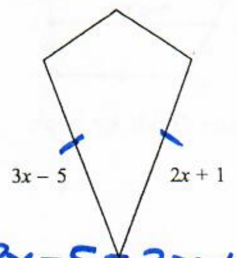
No parallel sides

Diagonals are perpendicular.



Examples:

Solve for x.



$3x-5=2x+1$

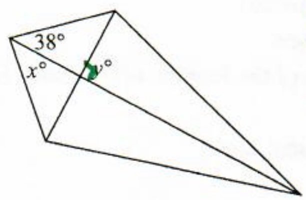
$-2x \quad -2x$

$x-5=1$

$+5 \quad +5$

$\boxed{x=6}$

Find the missing angle measures.



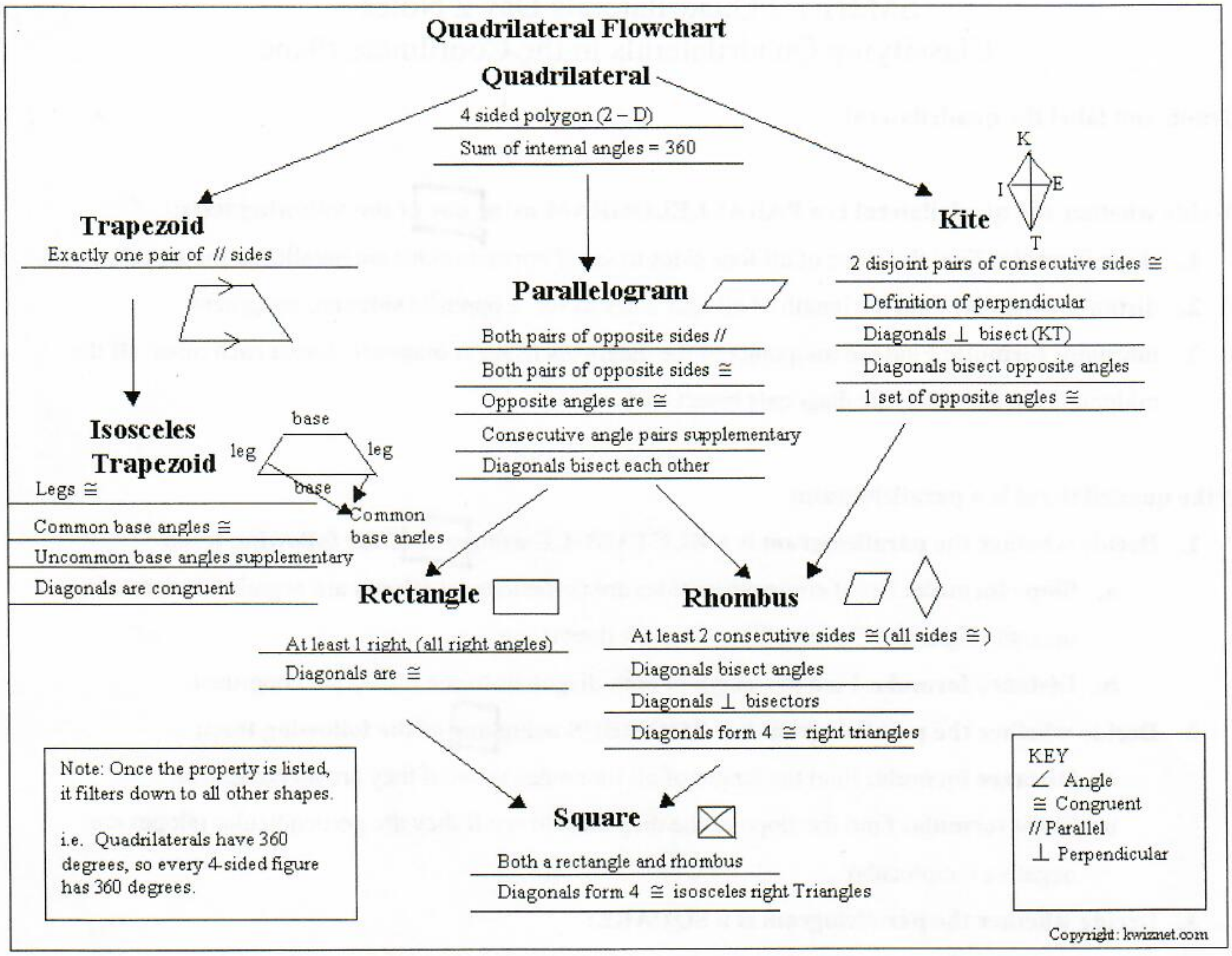
$x=38°$

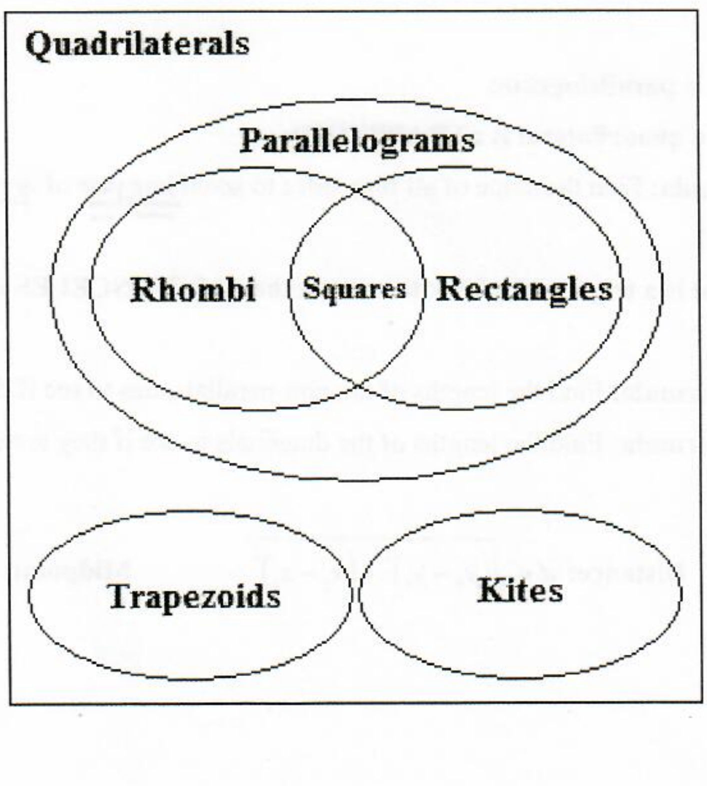
$y=90°$

Directions: Put an "x" in the box if the statement is always true for each type of quadrilateral.

	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles Trapezoid
Both pairs of opposite sides are parallel	x	x	x	x		
Diagonals are congruent		x		x		x
Both pairs of opposite angles are congruent	x	x	x	x		
Diagonals bisect each other	x	x	x	x		
All pairs of consecutive angles are supplementary	x	x	x	x		
Diagonals are perpendicular			x	x		
Exactly one pair of parallel sides					x	x
Both pairs of opposite sides are congruent	x	x	x	x		
All four sides are congruent			x	x		
Diagonals are angle bisectors			x	x		
Has four right angles		x		x		

kite


Quadrilateral Flowchart
Quadrilateral
4 sided polygon (2 – D)
Sum of internal angles = 360
Trapezoid
Exactly one pair of // sides
Isosceles Trapezoid
base
leg
leg
base
Common base angles
Legs ≅
Common base angles ≅
Uncommon base angles supplementary
Diagonals are congruent
Parallelogram
Both pairs of opposite sides //
Both pairs of opposite sides ≅
Opposite angles are ≅
Consecutive angle pairs supplementary
Diagonals bisect each other
Kite
K
I
E
T
2 disjoint pairs of consecutive sides ≅
Definition of perpendicular
Diagonals ⊥ bisect (KT)
Diagonals bisect opposite angles
1 set of opposite angles ≅
Rectangle
At least 1 right, (all right angles)
Diagonals are ≅
Rhombus
At least 2 consecutive sides ≅ (all sides ≅)
Diagonals bisect angles
Diagonals ⊥ bisectors
Diagonals form 4 ≅ right triangles
Note: Once the property is listed, it filters down to all other shapes.
i.e. Quadrilaterals have 360 degrees, so every 4-sided figure has 360 degrees.
Square
Both a rectangle and rhombus
Diagonals form 4 ≅ isosceles right Triangles
KEY
∠ Angle
≅ Congruent
// Parallel
⊥ Perpendicular
Copyright: kwiznet.com


Quadrilaterals
Parallelograms
Rhombi
Squares
Rectangles
Trapezoids
Kites

SM2H 7.7 Quadrilaterals Day 2 Notes – Classifying Quadrilaterals in the Coordinate Plane

Graph and label the quadrilateral

Decide whether the quadrilateral is a PARALLELOGRAM using one of the following tests:

1. **slope formula:** Find the slope of all four sides to see if opposite sides are parallel (same slope).
2. **distance formula:** Find the length of all four sides to see if opposite sides are congruent.
3. **midpoint formula:** Find the midpoints of the diagonals to see if diagonals bisect each other. (If the midpoints are the same, the diagonals bisect each other.)

If the quadrilateral is a parallelogram:

1. **Decide whether the parallelogram is a RECTANGLE using one of the following tests:**
   a. **Slope formula:** See if consecutive sides are perpendicular (slopes are negative reciprocals—opposite signs and fraction flipped upside down).
   b. **Distance formula:** Find the length of both diagonals to see if they are congruent.
2. **Decide whether the parallelogram is a RHOMBUS using one of the following tests:**
   a. **Distance formula:** Find the length of all four sides to see if they are all congruent.
   b. **Slope formula:** Find the slope of the diagonals to see if they are perpendicular (slopes are negative reciprocals).
3. **Decide whether the parallelogram is a SQUARE:**
   a. If the parallelogram is both a rhombus and a rectangle, then it is a square.

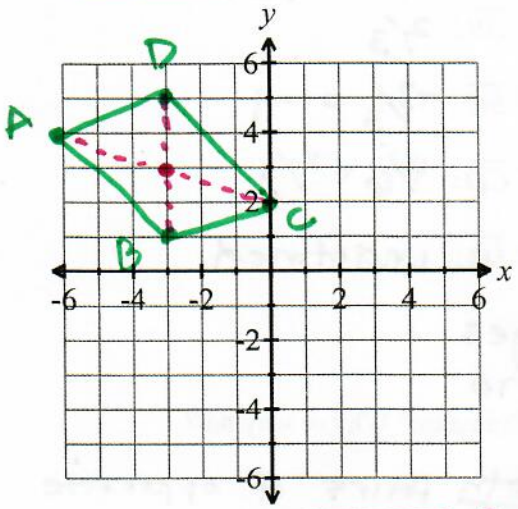
If the quadrilateral is not a parallelogram:

1. **Decide whether the quadrilateral is aTRAPEZOID:**
   a. **Slope Formula:** Find the slope of all four sides to see if one pair of opposite sides is parallel (same slope).
2. **If the quadrilateral is a trapezoid, check to see whether it is ISOSCELES using one of the following tests:**
   a. **Distance Formula:** Find the lengths of the non-parallel sides to see if they are congruent.
   b. **Distance Formula:** Find the lengths of the diagonals to see if they are congruent.

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ **Distance:** $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$ **Midpoint:** $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Decide whether the quadrilateral with vertices at $A(-6,4)$, $B(-3,1)$, $C(0,2)$, and $D(-3,5)$ is a parallelogram.

Graph and label the quadrilateral



$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Method 1: Slope Formula $\frac{y_2-y_1}{x_2-x_1}$

Slope of $\overline{AB}$: $\frac{1-4}{-3+6} = \frac{-3}{3} = -1$

Slope of $\overline{BC}$: $\frac{2-1}{0--3} = \frac{1}{3}$

Slope of $\overline{CD}$: $\frac{5-2}{-3-0} = \frac{3}{-3} = -1$

Slope of $\overline{DA}$: $\frac{4-5}{-6+3} = \frac{-1}{-3} = \frac{1}{3}$

Is $\overline{AB} \parallel \overline{CD}$? yes

Is $\overline{BC} \parallel \overline{DA}$? yes

Is *ABCD* a parallelogram? Why or why not?

yes, <u>both</u> pairs of opposite sides are parallel.

Method 2: Distance Formula

Length of $\overline{AB}$: $\sqrt{(-3+6)^2+(1-4)^2} = \sqrt{3^2+(-3)^2} = \sqrt{18}$

Length of $\overline{BC}$: $\sqrt{(0--3)^2+(2-1)^2} = \sqrt{3^2+1^2} = \sqrt{10}$

Length of $\overline{CD}$: $\sqrt{(-3-0)^2+(5-2)^2} = \sqrt{(-3)^2+3^2} = \sqrt{18}$

Length of $\overline{DA}$: $\sqrt{(-6+3)^2+(4-5)^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{10}$

Is $\overline{AB} \cong \overline{CD}$? yes

Is $\overline{BC} \cong \overline{DA}$? yes

Is *ABCD* a parallelogram? Why or why not?

yes, <u>both</u> pairs of opposite sides are congruent

Method 3: Midpoint Formula midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint of $\overline{AC}$:

$\left(\frac{-6+0}{2}, \frac{4+2}{2}\right) = (-3, 3)$

Midpoint of $\overline{BD}$:

$\left(\frac{-3+-3}{2}, \frac{1+5}{2}\right) = (-3, 3)$

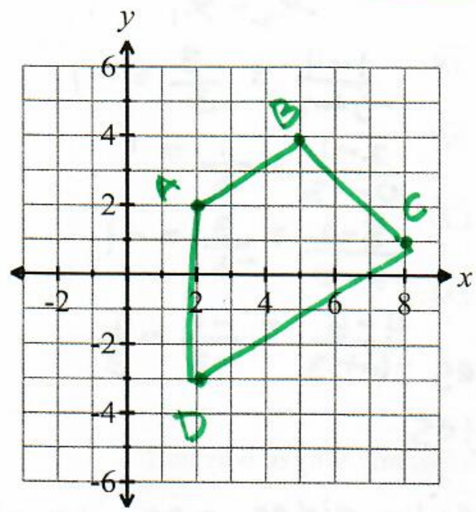
Are the midpoints of the diagonals the same? yes

Is *ABCD* a parallelogram? Why or why not?

yes, the diagonals bisect each other.

Decide whether the quadrilateral with vertices at $A(2,2)$, $B(5,4)$, $C(8,1)$, and $D(2,-3)$ is a parallelogram.

Graph and label the quadrilateral



Method 1: Slope Formula count slope $\frac{\text{rise}}{\text{run}}$

Slope of $\overline{AB}$: $2/3$

Slope of $\overline{BC}$: $-3/3 = -1$

Slope of $\overline{CD}$: $4/6 = 2/3$

Slope of $\overline{DA}$: undefined

Is $\overline{AB} \parallel \overline{CD}$? yes

Is $\overline{BC} \parallel \overline{DA}$? no

Is *ABCD* a parallelogram? Why or why not?

no, both pairs of opposite sides are not parallel.

Method 2: Distance Formula

Length of $\overline{AB}$: $\sqrt{(5-2)^2+(4-2)^2} = \sqrt{3^2+2^2} = \sqrt{13}$

Length of $\overline{BC}$: $\sqrt{(8-5)^2+(1-4)^2} = \sqrt{3^2+(-3)^2} = \sqrt{18}$

Length of $\overline{CD}$: $\sqrt{(2-8)^2+(-3-1)^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{52}$

Length of $\overline{DA}$: $\sqrt{(2-2)^2+(2--3)^2} = \sqrt{0^2+5^2} = \sqrt{25}$

Is $\overline{AB} \cong \overline{CD}$? no

Is $\overline{BC} \cong \overline{DA}$? no

Is *ABCD* a parallelogram? Why or why not?

no, both pairs of opposite sides are not parallel.

Method 3: Midpoint Formula

Midpoint of $\overline{AC}$: $\left(\frac{2+8}{2}, \frac{2+1}{2}\right) = \left(5, \frac{3}{2}\right)$

Midpoint of $\overline{BD}$: $\left(\frac{5+2}{2}, \frac{4+-3}{2}\right) = \left(\frac{7}{2}, \frac{1}{2}\right)$

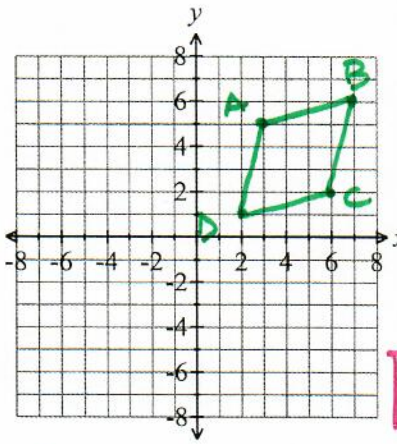
Are the midpoints of the diagonals the same? no

Is *ABCD* a parallelogram? Why or why not?

no, the diagonals do not bisect each other.

Graph and label each quadrilateral with the given vertices. Then determine the most precise name for each quadrilateral. You may use any tests you want, but you must show all your work!

1. $A(3,5)$, $B(7,6)$, $C(6,2)$, $D(2,1)$

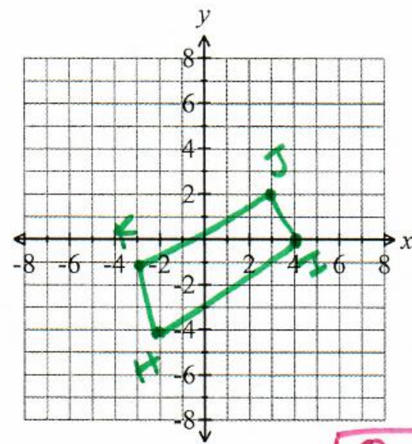


slopes: ✓
AB 1/4
BC 4
CD 1/4
DA 4
·parallel sides

$\boxed{\text{rhombus}}$

distance: AB $\sqrt{(7-3)^2+(6-5)^2}$ ✓
$=\sqrt{4^2+1^2}=\sqrt{17}$
BC $\sqrt{(6-7)^2+(2-6)^2}$
$=\sqrt{(-1)^2+(-4)^2}=\sqrt{17}$
CD $\sqrt{(2-6)^2+(1-2)^2}$
$=\sqrt{(-4)^2+(-1)^2}=\sqrt{17}$
DA $=\sqrt{(3-2)^2+(5-1)^2}=\sqrt{1^2+4^2}=\sqrt{17}$

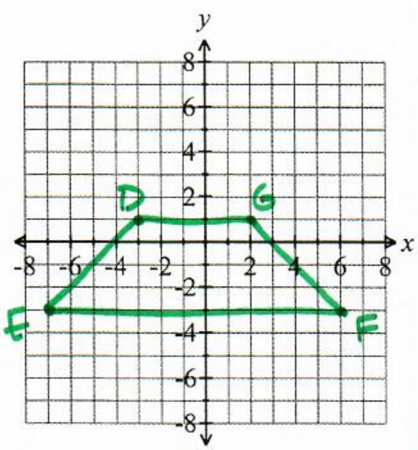
2. $H(-2,-3)$, $I(4,0)$, $J(3,2)$, $K(-3,-1)$



Slopes
KJ $3/6=1/2$
JI -2
HI $4/6=2/3$
HK -3

$\boxed{\text{Quadrilateral}}$

3. $D(-3,1)$, $E(-7,-3)$, $F(6,-3)$, $G(2,1)$



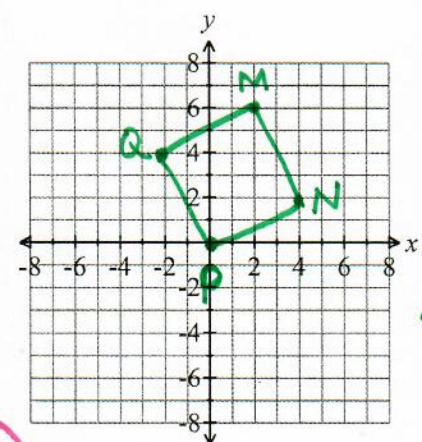
Slopes: DG $0/5=0$ ✓
EF $0/13=0$

$\boxed{\text{isosceles trapezoid}}$

trapezoid – check for isosceles trapezoid

distance: DE $\sqrt{(-7+3)^2+(-3-1)^2}$ ✓
$=\sqrt{(-4)^2+(-4)^2}=\sqrt{32}$
GF $\sqrt{(2-6)^2+(1--3)^2}=\sqrt{32}$

4. $M(2,6)$, $N(4,2)$, $P(0,0)$, $Q(-2,4)$



Slopes:
QM $2/4=1/2$
MN $-4/2=-2$
NP $2/4=1/2$
PQ $-4/2=-2$
·4 right angles

distance ✓ QM $\sqrt{(-2-2)^2+(4-6)^2}$
$=\sqrt{(-4)^2+(-2)^2}=\sqrt{20}$
MN $\sqrt{(4-2)^2+(2-6)^2}$
$=\sqrt{2^2+(-4)^2}=\sqrt{20}$
NP $\sqrt{(0-4)^2+(0-2)^2}$
$=\sqrt{(-4)^2+(-2)^2}=\sqrt{20}$
PQ $\sqrt{(-2-0)^2+(4-0)^2}$
$=\sqrt{(-2)^2+4^2}=\sqrt{20}$

$\boxed{\text{Square}}$