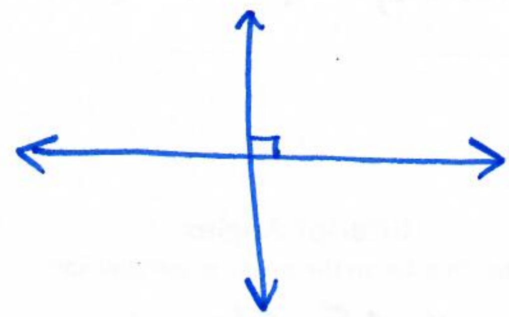
Name:____ Period:__________

SM2H Unit 7 Notes

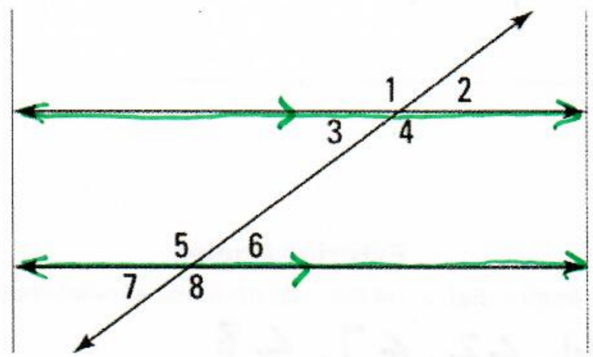
7.1 Parallel Lines and Angles

Perpendicular Lines
2 lines that intersect to make a right angle.

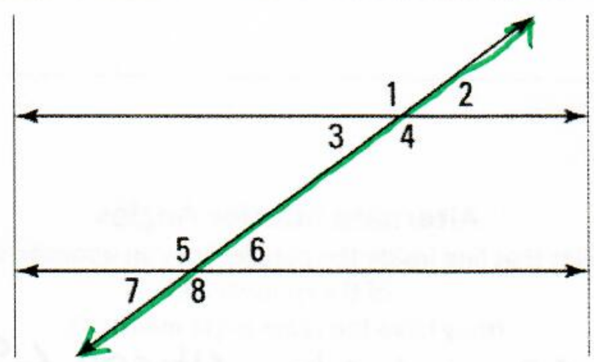
Draw a picture of perpendicular lines here.



Parallel Lines
2 lines that never intersect

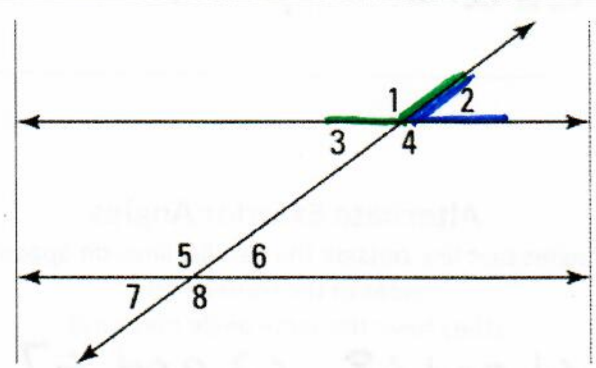


Transversal
A line that cuts a set of parallel lines



Supplementary Angles
2 angles that add up to 180 degrees

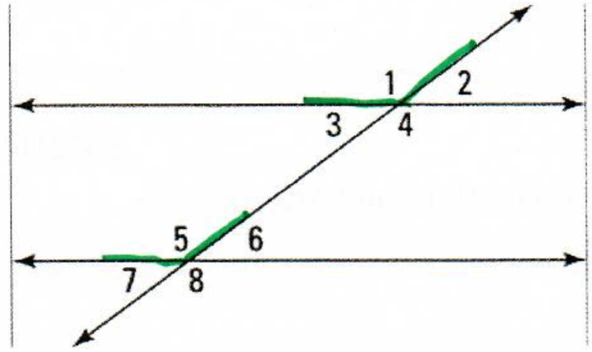
∠1 and ∠2, ∠1 and ∠3,
∠2 and ∠4, ∠3 and ∠4,
∠5 and ∠6, ∠7 and ∠8,
∠5 and ∠7, ∠6 and ∠8



Corresponding Angles

2 angles that lie in the same position on each parallel line while touching the transversal
(they have the same angle measure)

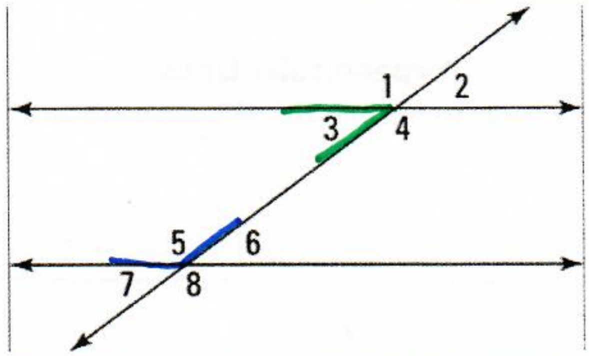
$\angle 1$ and $\angle 5$, $\angle 3$ and $\angle 7$,
$\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$



Interior Angles

Angles that lie on the inside of parallel lines

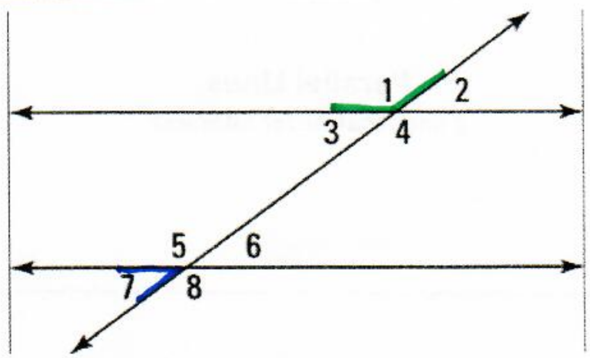
$\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$



Exterior Angles

Angles that lie on the outside of the parallel lines

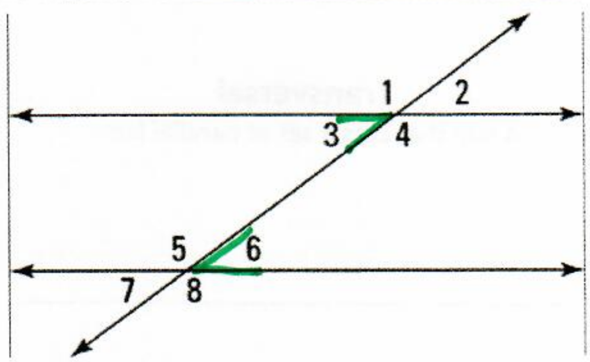
$\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$



Alternate Interior Angles

Angles that line inside the parallel lines on opposite sides of the transversal
(they have the same angle measure)

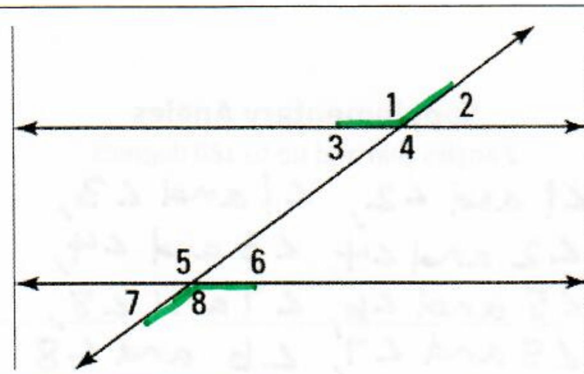
$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$

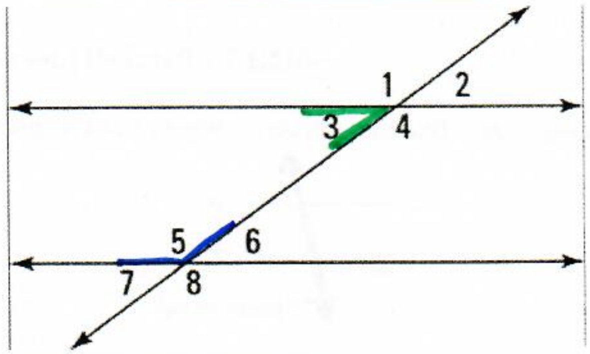
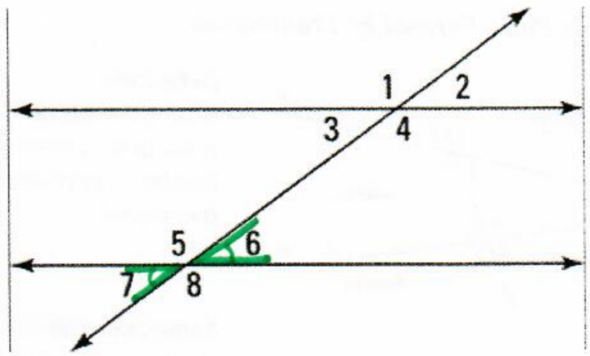
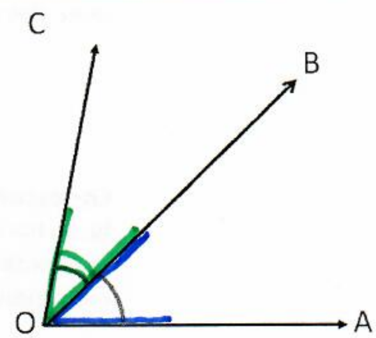
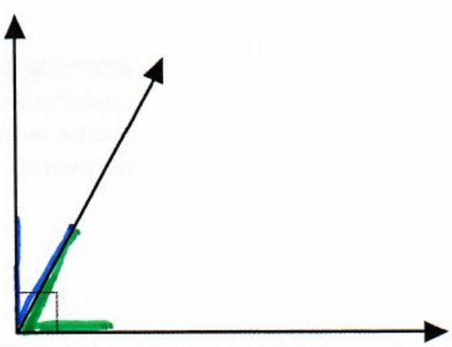


Alternate Exterior Angles

Angles that line outside the parallel lines on opposite sides of the transversal
(they have the same angle measure)

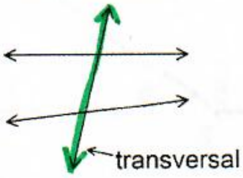
$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$



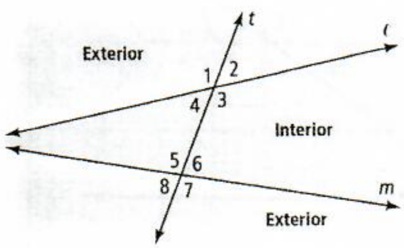
Same-Side Interior Two angles that are on the same side of the transversal and inside the parallel lines (they are supplementary) ∠3 and ∠5, ∠4 and ∠6	
Vertical Angles A pair of opposite angles made by two intersecting lines (they have the same angle measure) ∠1 and ∠4, ∠2 and ∠3, ∠5 and ∠8, ∠6 and ∠7	
Adjacent Angles Two angles that have a common vertex and a common side	
Complementary Angles 2 angles that add to 90 degrees	

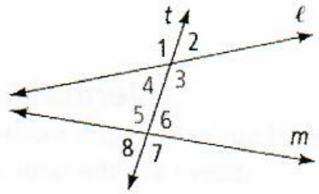
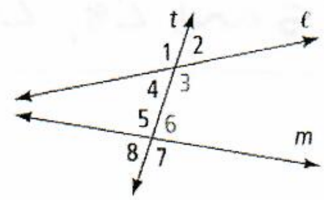
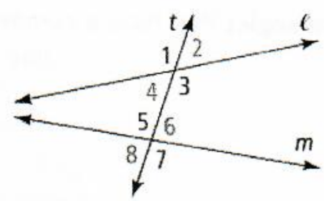
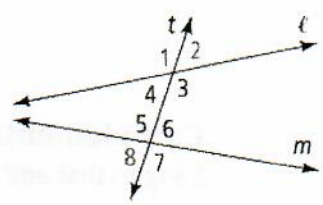
SM2H 7.2 Parallell Lines and Angle Relationships Notes

Transversal: A line that intersects two or more coplanar lines at different points.

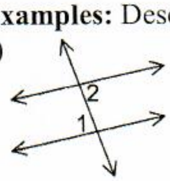


Angle Pairs Formed by Transversals:

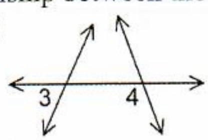


Definition	Example	
Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal.	∠4 and ∠6 ∠3 and ∠5	
Same-side interior angles are interior angles that lie on the same side of the transversal.	∠4 and ∠5 ∠3 and ∠6	
Corresponding angles lie on the same side of the transversal *t* and in corresponding positions.	∠1 and ∠5 ∠4 and ∠8 ∠2 and ∠6 ∠3 and ∠7	
Alternate exterior angles are nonadjacent exterior angles that lie on opposite sides of the transversal.	∠1 and ∠7 ∠2 and ∠8	

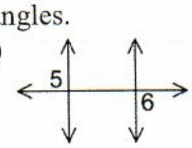
Examples: Describe the relationship between the numbered angles.

a)


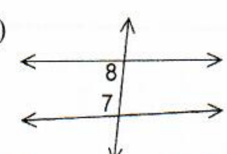
alternate interior

b)


Corresponding

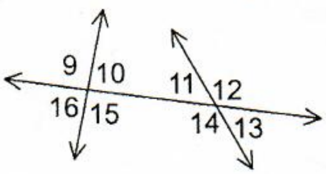
c)


alternate exterior

d)


Same-side interior

Examples: List all pairs of angles that fit the description.



a) alternate exterior

∠9 and ∠13
∠16 and ∠12

b) corresponding

∠9 and ∠11, ∠10 and ∠12,
∠16 and ∠14, ∠15 and ∠13

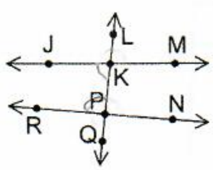
c) same-side interior

∠10 and ∠11
∠15 and ∠14

d) alternate interior

∠15 and ∠11
∠10 and ∠14

Examples: Describe the relationship between each pair of angles.



a) $\angle JKP$ and $\angle KPN$

alternate interior

b) $\angle LKM$ and $\angle QPR$

alternate exterior

c) $\angle JKL$ and $\angle RPK$

corresponding

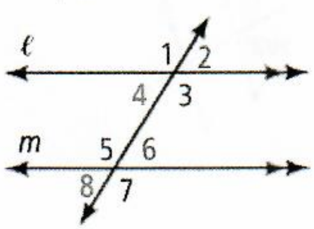
d) $\angle JKP$ and $\angle KPR$

same-side interior

Corresponding Angles Postulate: If a transversal intersects two parallel lines, then corresponding angles are congruent.

If . . .

$\ell \parallel m$



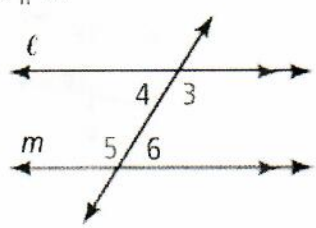
Then . . .

$\angle 1 \cong \angle 5$
$\angle 2 \cong \angle 6$
$\angle 3 \cong \angle 7$
$\angle 4 \cong \angle 8$

Alternate Interior Angles Theorem: If a transversal intersects two parallel lines, then alternate interior angles are congruent.

If . . .

$\ell \parallel m$



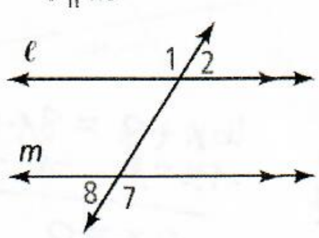
Then . . .

$\angle 4 \cong \angle 6$
$\angle 3 \cong \angle 5$

Alternate Exterior Angles Theorem: If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

If . . .

$\ell \parallel m$



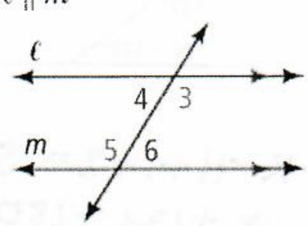
Then . . .

$\angle 1 \cong \angle 7$
$\angle 2 \cong \angle 8$

Same-Side Interior Angles Theorem: If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

If . . .

$\ell \parallel m$

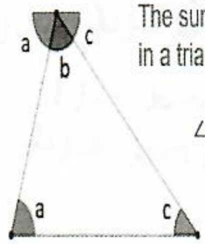


Then . . .

$m\angle 4 + m\angle 5 = 180$
$m\angle 3 + m\angle 6 = 180$

Other Useful Theorems:

Triangle Sum Theorem



The sum of the three interior angles in a triangle is always 180°.

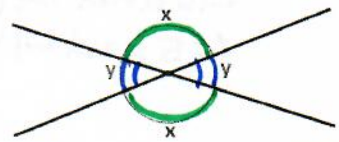
$\angle a + \angle b + \angle c = 180°$

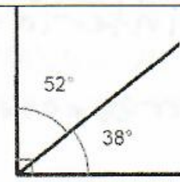
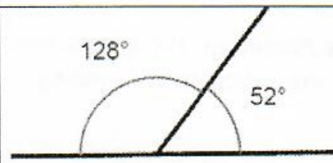
Vertical Angles

Vertical Angles are pairs of opposite angles made by intersecting lines.

Vertical Angle Theorem

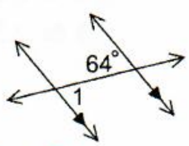
If 2 angles are vertical then they are congruent.



Type of Angles	Description	Example
Complementary Angles	Angles that add up to 90°	
Supplementary Angles	Angles that add up to 180°	

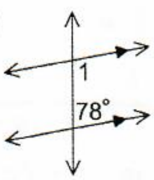
Examples: Find $m\angle 1$ in each diagram. Give a reason for each answer.

a)



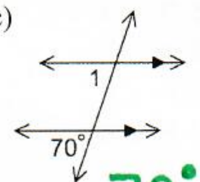
64°
alternate interior angles are ≅

b)



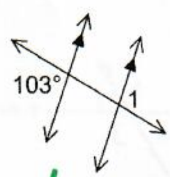
102°
Same-side interior angles are supplementary

c)



70°
corresponding angles are ≅

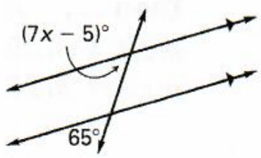
d)



103°
alternate exterior angles are congruent

Find the value of x. Give a reason for each answer.

e)



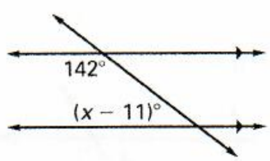
$65 = 7x - 5$
$+5 \quad\quad +5$

$\frac{70}{7} = \frac{7x}{7}$

$10 = x$

Corresponding angles are ≅.

f)



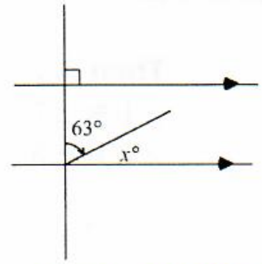
$x - 11 + 142 = 180$

$x + 131 = 180$
$-131 \quad -131$

$x = 49$

Same-side interior angles are supplementary.

g)

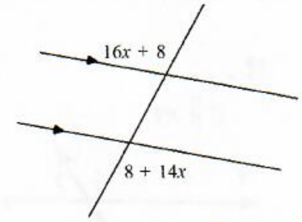


$x + 63 = 90$
$-63 \quad -63$

$x = 27$

Complementary angles

h)



$16x + 8 = 8 + 14x$
$-14x - 8 \quad -8 - 14x$

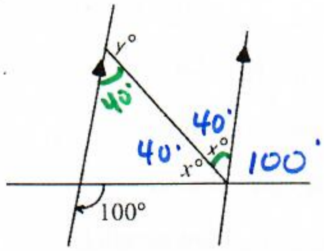
$2x = 0$

$x = 0$

alternate exterior angles are congruent

Find the values of x and y.

a)



$x + x + 100 = 180$
$-100 \quad -100$

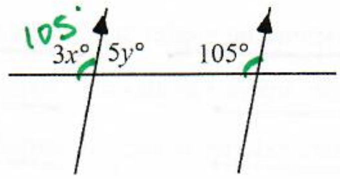
$2x = 80$

$\boxed{x = 40}$

$y + 40 = 180$

$\boxed{y = 140}$

b)



$\frac{3x}{3} = \frac{105}{3}$

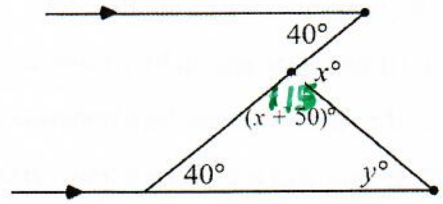
$\boxed{x = 35}$

$105 + 5y = 180$

$5y = 75$

$\boxed{y = 15}$

c)



$x + x + 50 = 180$

$2x = 130$

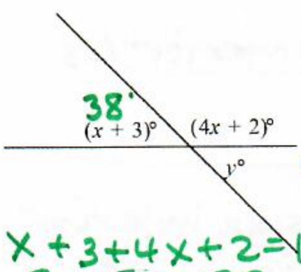
$\boxed{x = 65}$

$115 + 40 + y = 180$

$155 + y = 180$

$\boxed{y = 25}$

d)



$y = x + 3$

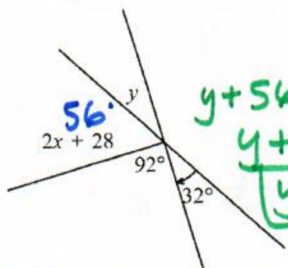
$\boxed{y = 38}$

$x + 3 + 4x + 2 = 180$

$5x + 5 = 180$

$5x = 175$ $\boxed{x = 35}$

e)



$y + 56 + 92 = 180$

$y + 148 = 180$

$\boxed{y = 32}$

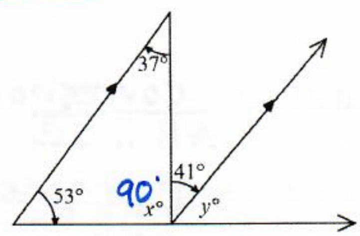
$92 + 32 + 2x + 28 = 180$

$152 + 2x = 180$

$2x = 28$

$\boxed{x = 14}$

f)



$x + 53 + 37 = 180$

$x + 90 = 180$

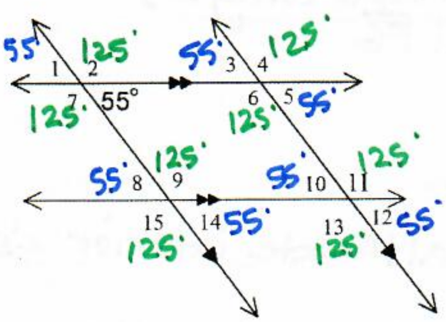
$\boxed{x = 90}$

$90 + 41 + y = 180$

$131 + y = 180$

$\boxed{y = 49}$

g) Find the measure of each numbered angle.



$m\angle 1 = 55°$

$m\angle 2 = 125°$

$m\angle 3 = 55°$

$m\angle 4 = 125°$

$m\angle 5 = 55°$

$m\angle 6 = 125°$

$m\angle 7 = 125°$

$m\angle 8 = 55°$

$m\angle 9 = 125°$

$m\angle 10 = 55°$

$m\angle 11 = 125°$

$m\angle 12 = 55°$

$m\angle 13 = 125°$

$m\angle 14 = 55°$

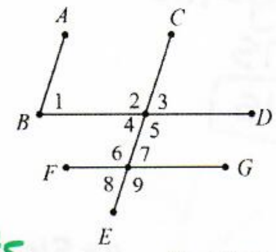
$m\angle 15 = 125°$

SM2H 7.3 Proving Lines Parallel Notes

Ways to Prove Lines are Parallel:

1. If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.
2. If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.
3. If two lines are cut by a transversal and alternate exterior angles are congruent, then the lines are parallel.
4. If two lines are cut by a transversal and same side interior angles are supplementary, then the lines are parallel.

Given the following figure, determine which set of line segments must be parallel given the following information. If no lines are parallel say none. **State the postulate or theorem that justifies your answer.**



a) $\angle 1 \cong \angle 3$ Corresponding ∠'s $\overline{AB} \parallel \overline{CE}$

b) $\angle 2 \cong \angle 9$ alternate exterior ∠'s $\overline{BD} \parallel \overline{FG}$

c) $\angle 4 \cong \angle 3$ Vertical ∠'s none

d) $\angle 5 \cong \angle 6$ alternate interior ∠'s $\overline{BD} \parallel \overline{FG}$

e) $m\angle 1 + m\angle 2 = 180$ same-side interior ∠'s $\overline{AB} \parallel \overline{CE}$

f) $\angle 1 \cong \angle 4$ alternate interior ∠'s $\overline{AB} \parallel \overline{CE}$

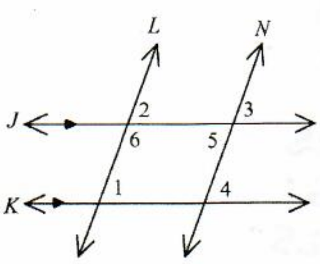
g) $\angle 1 \cong \angle 5$ none

h) $m\angle 4 + m\angle 6 = 180$ Same-side interior ∠'s $\overline{BD} \parallel \overline{FG}$

i) $m\angle 4 + m\angle 2 = 180$ none

j) $\angle 5 \cong \angle 9$ Corresponding ∠'s $\overline{BD} \parallel \overline{FG}$

Which pairs of angles, if shown congruent or supplementary, would give $L \parallel N$?



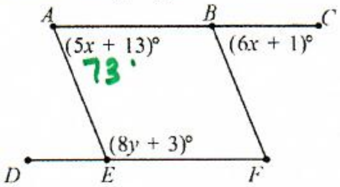
k) If $\angle$ 2 $\cong \angle$ 5, then $L \parallel N$ by alternate interior ∠'s

l) If $\angle$ 1 $\cong \angle$ 4, then $L \parallel N$ by Corresponding ∠'s.

m) If $\angle$ 2 $\cong \angle$ 3, then $L \parallel N$ by Corresponding ∠'s.

n) If $\angle$ 6 is supplementary to $\angle$ 5, then $L \parallel N$ by Same-side interior ∠'s.

Use the following figure to answer a and b.



a) Find the value of x that makes $\overline{AE} \parallel \overline{BF}$.

$5x + 13 = 6x + 1$
$-5x \quad\quad -5x$
$13 = x + \cancel{1}$
$-1 \quad\quad \cancel{-1}$
$\boxed{12 = x}$

Corresponding ∠'s are ≅

What theorem did you use to show that $\overline{AE} \parallel \overline{BF}$?

b) Find the value of y that makes $\overline{AC} \parallel \overline{DF}$.

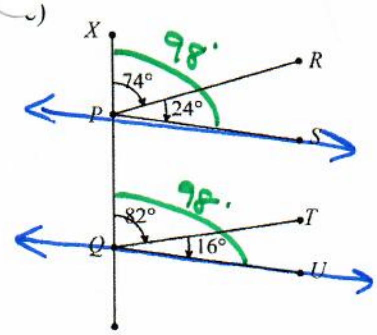
$73 + 8y + 13 = 180$
$86 + 8y = 180$
$\cancel{-86} \quad -86$
$\frac{8y}{8} = \frac{94}{8}$
$\boxed{y = 11.75}$

Same-side interior ∠'s are supplementary

What theorem did you use to show that $\overline{AC} \parallel \overline{DF}$?

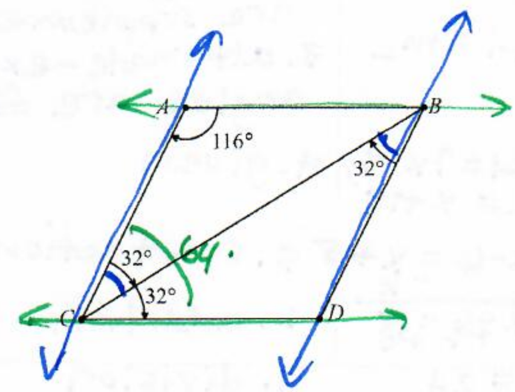
Which segments, if any, are parallel?

c)



$\overline{PS} \parallel \overline{QU}$

d)



$\overline{AB} \parallel \overline{CD}$, same-side interior angles are supplementary.

$\overline{AC} \parallel \overline{BD}$, alternate interior ∠'s are congruent.

Explain:

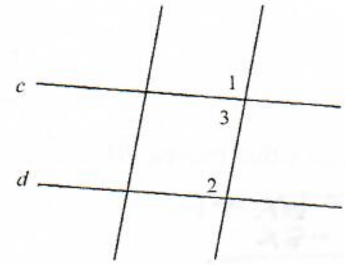
Corresponding ∠'s are ≅

Explain:

See explanations above.

1. Given $\angle 3$ & $\angle 2$ are supplementary, prove $\angle 1 \cong \angle 2$.

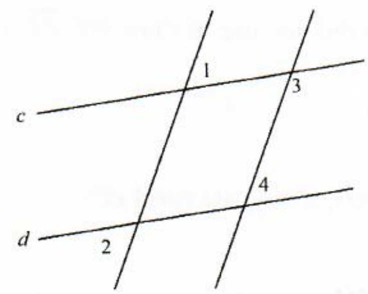
Statements	Reasons
1. $\angle 3$ and $\angle 2$ are supplementary	1. given
2. $c \parallel d$	2. same-side interior angles are supplementary
3. $\angle 1 \cong \angle 2$	3. corresponding angles are $\cong$.



2. Given m$\angle 3$ + $\angle 4$ +180 and m$\angle 1$= $(7x-6)^\circ$, m$\angle 2$= $(x+18)^\circ$. Prove ~~x = 12~~ $x=4$. Hint: You must prove the c & d are parallel.

Statements	Reasons
1. $m\angle 3 + m\angle 4 = 180$	1. given
2. $c \parallel d$	2. Same-side interior $\angle$'s are supplementary
3. $\angle 1 \cong \angle 2$	3. alternate-exterior angles are $\cong$.
4. $m\angle 1 = 7x-6$, $m\angle 2 = x+18$	4. given
5. $7x-6 = x+18$; $-x$ $-x$	5. subtraction
6. $6x - 6 = 18$; $+6$ $+6$; $6x = 24$	6. addition
7. $x = 4$	7. division

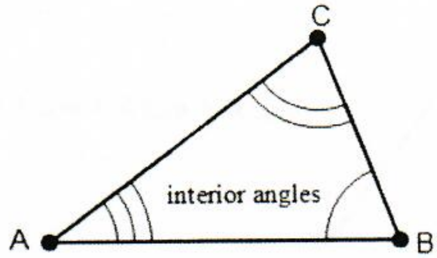
≅



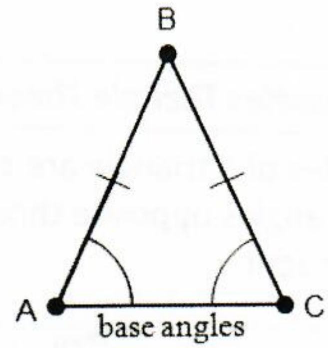
SM2H 7.4 Triangle Notes

VOCABULARY

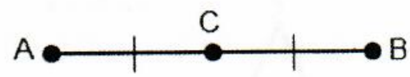
The angles inside of a triangle are called **interior angles**.



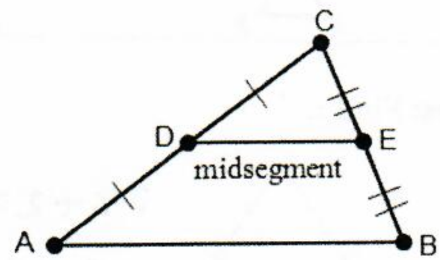
A triangle with at least two congruent sides is called an **isosceles triangle**. In an isosceles triangle, the angles that are opposite the congruent sides are called **base angles**.



A point that is halfway between the endpoints of a segment is called the **midpoint**. Point C is the midpoint of $\overline{AB}$.

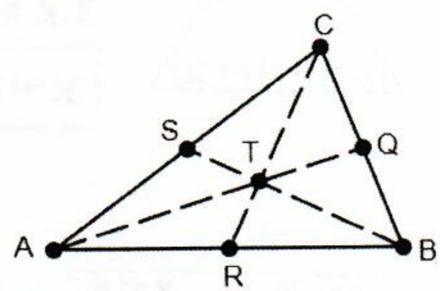


A segment whose endpoints are the midpoints of two sides of a triangle is called the **midsegement of a tiangle**.



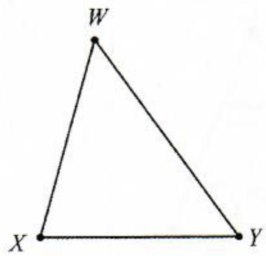
The line connecting midpoints to the opposite vertex of a triangle is called the **median**. Point S is the midpoint of $\overline{AC}$. Point Q is the midpoint of $\overline{BC}$. Point R is the midpoint of $\overline{AB}$.

The point where all three medians of a triangle intersect is called a **centroid**. Point T is the centroid of $\triangle ABC$.



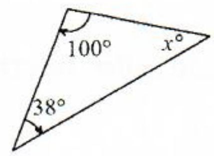
Other Triangle Theorems:

Angle Sum Theorem: The sum of the measures of the angles of a triangle is 180°.



$m\angle W + m\angle X + m\angle Y = 180°$

Example: Find x.



$x + 100 + 38 = 180$

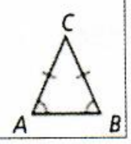
$x + 138 = 180$

$x = 42$

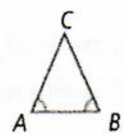
Isosceles Triangle Theorem	*Converse of the Isosceles Triangle Theorem*
If two sides of a triangle are congruent, then the angles opposite those sides are congruent	If two angles of a triangle are congruent , then the sides opposite those angles are congruent

If . . . $\overline{AC} \cong \overline{BC}$

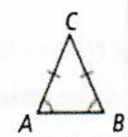
Then . . . $\angle A \cong \angle B$



If . . . $\angle A \cong \angle B$

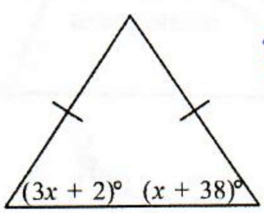


Then . . . $\overline{AC} \cong \overline{BC}$



Examples: Find x.

a)

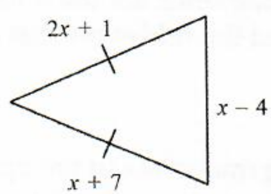


$3x + 2 = x + 38$

$2x = 40$

$\boxed{x = 20}$

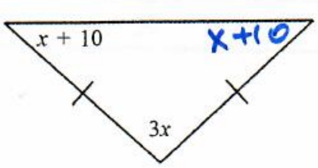
b)



$2x + 1 = x + 7$

$\boxed{x = 6}$

c)



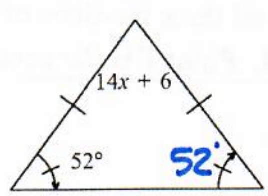
$x + 10 + x + 10 + 3x = 180$

$5x + 20 = 180$

$5x = 160$

$\boxed{x = 32}$

d)



$52 + 52 + 14x + 6 = 180$

$14x + 110 = 180$

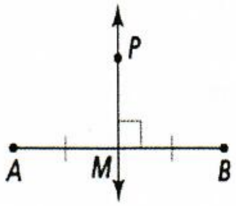
$14x = 70$

$\boxed{x = 5}$

Perpendicular Bisector Theorem

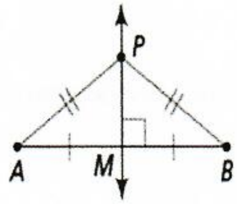
If . . .

$\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$



Then . . .

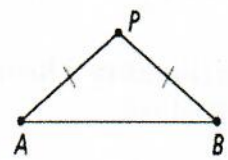
$PA = PB$



Converse of Perpendicular Bisector Theorem

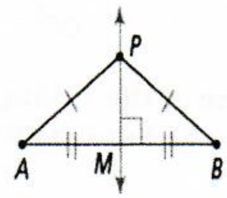
If . . .

$PA = PB$

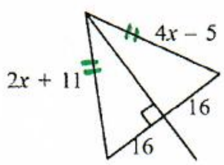


Then . . .

$\overleftrightarrow{PM} \perp \overline{AB}$ and $MA = MB$



a) Find x.

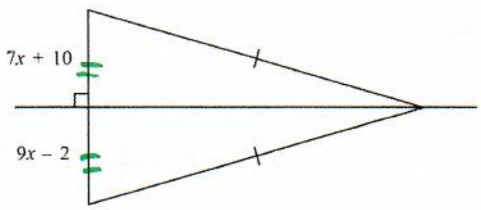


$2x+11=4x-5$

$16=2x$

$\boxed{8=x}$

b) Find x.

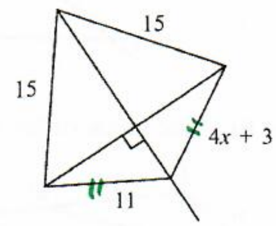


$7x+10=9x-2$

$12=2x$

$\boxed{6=x}$

c) Find x.

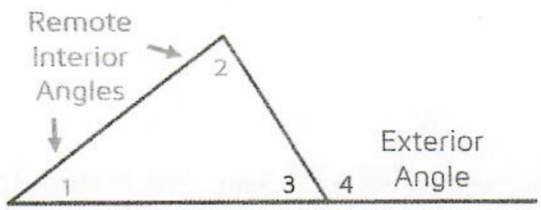


$4x+3=11$

$4x=8$

$\boxed{x=2}$

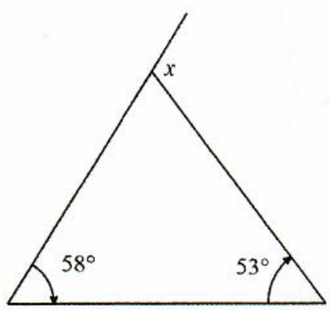
Exterior Angle Theorem: The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.



Find x.

$m\angle 1 + m\angle 2 = m\angle 4$

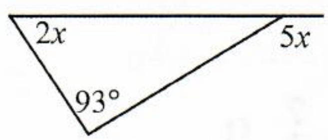
d)



$x=58+53$

$\boxed{x=111°}$

e)

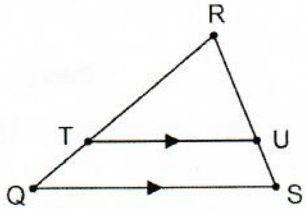


$5x=2x+93$

$3x=93$

$\boxed{x=31}$

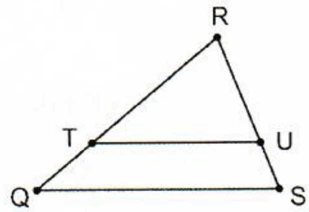
Triangle Proportionality Theorem: If a line parallel to one side of a triangle intersects the other two sides, then it divides the sides proportionally.



In $\triangle QRS$, if $\overline{TU} \parallel \overline{QS}$,

then $\dfrac{RT}{TQ} = \dfrac{RU}{US}$.

Converse of the Triangle Proportionality Theorem*:* If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

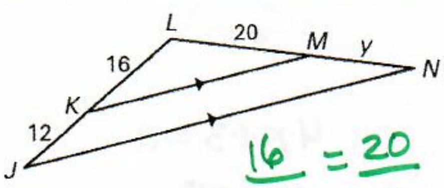


In $\triangle QRS$, if $\dfrac{RT}{TQ} = \dfrac{RU}{US}$,

then $\overline{TU} \parallel \overline{QS}$.

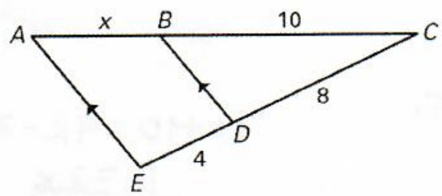
Examples: Find the value of the variable.

a)



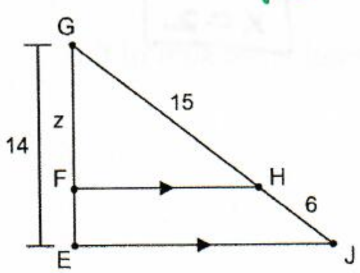
$\dfrac{16}{12} = \dfrac{20}{y}$ $\quad 16y = 32$ $\quad \boxed{y = 2}$

b)



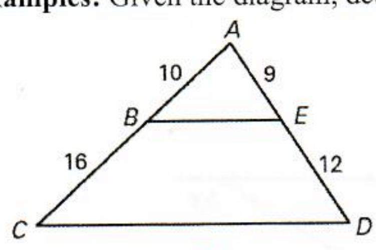
$\dfrac{10}{x} = \dfrac{8}{4}$ $\quad 40 = 8x$ $\quad \boxed{5 = x}$

c)



Examples: Given the diagram, determine whether $\overline{BE} \parallel \overline{CD}$. Show work to support your answer.

a)

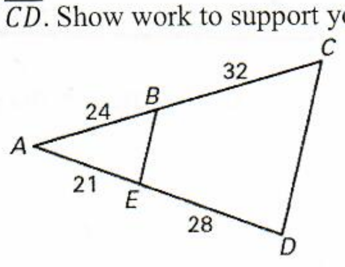


$\dfrac{10}{16} \stackrel{?}{=} \dfrac{9}{12}$

$120 \neq 144$

$\overline{BE}$ is not parallel to $\overline{CD}$

b)



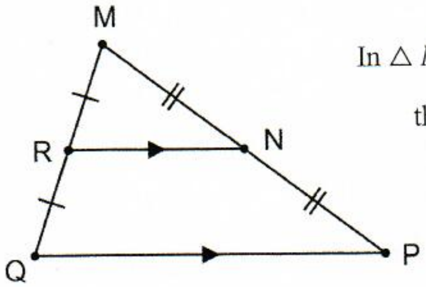
$\dfrac{24}{32} \stackrel{?}{=} \dfrac{21}{28}$

$672 = 672$

$\overline{BE} \parallel \overline{CD}$

Midsegment of a Triangle: A segment that connects the midpoints of two sides of a triangle.

idsegment Theorem: The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

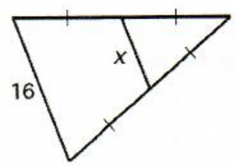


In $\triangle MPQ$, if $MR = RQ$ and $MN = NP$,

then $\overline{RN} \parallel \overline{QP}$ and $RN = \frac{1}{2}QP$.

Examples: Find the value of the variable.

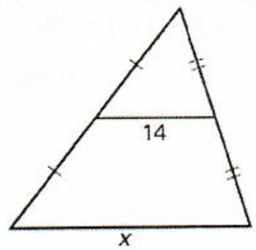
a)



$x = \frac{1}{2} \cdot 16$

$\boxed{x = 8}$

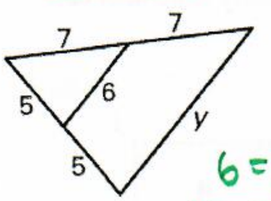
b)



$14 = \frac{1}{2} \cdot x$

$\boxed{28 = x}$

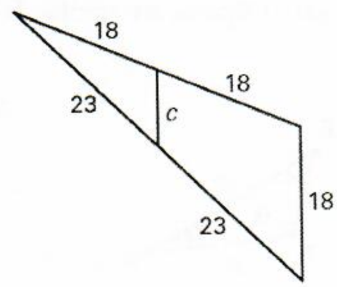
)



$6 = \frac{1}{2} y$

$\boxed{12 = y}$

d)

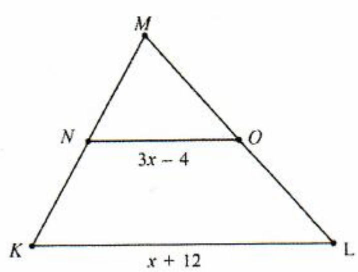


$c = \frac{1}{2} \cdot 18$

$\boxed{c = 9}$

Given that $\overline{NO}$ is a midsegment of the triangle, find x.

e)



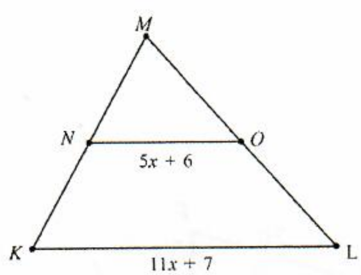
$3x - 4 = \frac{1}{2}(x + 12)$

$6x - 8 = x + 12$

$5x = 20$

$\boxed{x = 4}$

f)



$5x + 6 = \frac{1}{2}(11x + 7)$

$10x + 12 = 11x + 7$

$\boxed{5 = x}$