SM 2

Date:

Section: 9.1

Objective: Solving Proportions

Ratio:
A relationship between two quantities, normally expressed as the quotient of one divided by the other.

Proportion:
A statement that two ratios are equal.

Cross-Product Property:
Cross multiply – Used to solve a proportion.

Solve each proportion.

a. $\frac{15}{9} = \frac{10}{x}$

$15x = 90$

$\frac{15x}{15} = \frac{90}{15}$

$\boxed{x = 6}$

b. $\frac{7}{10} = \frac{a}{4}$

$28 = 10a$

$\frac{28}{10} = \frac{10a}{10}$

$\boxed{2.8 = a}$

c. $\frac{9}{6} = \frac{m}{3}$

$27 = 6m$

$\frac{27}{6} = \frac{6m}{6}$

$\boxed{4.5 = m}$

d. $\frac{8}{7} = \frac{k}{10}$

$80 = 7k$

$\frac{80}{7} = \frac{7k}{7}$

$\boxed{11.43 = k}$

e. $\frac{2}{x-1} = \frac{4}{8}$

$16 = 4(x-1)$

$16 = 4x - 4$

$+4 \quad +4$

$\frac{20}{4} = \frac{4x}{4}$

$\boxed{5 = x}$

f. $\frac{k+5}{6} = \frac{2}{3}$

$3(k+5) = 12$

$3k + 15 = 12$

$-15 \quad -15$

$\frac{3k}{3} = \frac{-3}{3}$

$\boxed{k = -1}$

g. $\frac{8}{2x+5} = \frac{5}{3}$

$24 = 5(2x+5)$

$24 = 10x + 25$

$-25 \quad -25$

$\frac{-1}{10} = \frac{10x}{10}$

$\boxed{-0.1 = x}$

h. $\frac{2}{9} = \frac{4}{3x+2}$

$2(3x+2) = 36$

$6x + 4 = 36$

$-4 \quad -4$

$\frac{6x}{6} = \frac{32}{6}$

$\boxed{x = 5.\overline{3}}$

Solve each problem using a proportion. Show your work.

a. The money used in Western Samoa is called the Tala. The exchange rate is 17 Tala to \$6. How many dollars would you receive if you exchanged 51 Tala?

$\frac{17 \text{ tala}}{\$6} = \frac{51 \text{ tala}}{x}$

$\frac{17x}{17} = \frac{306}{17}$

$\boxed{x = \$18}$

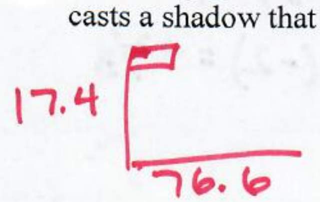
b. A model satellite has a scale of 3 cm: 2 m. If the model satellite is 24 cm wide, then how wide is the real satellite?

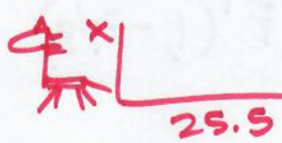
$\frac{3 \text{ cm}}{2 \text{ m}} = \frac{24 \text{ cm}}{x}$

$\frac{3x}{3} = \frac{48}{3}$

$\boxed{x = 16 \text{ m}}$

c. A baby giraffe standing near a flagpole casts a shadow that is 25.5 ft. long. If the 17.4-ft.-tall flagpole casts a shadow that is 76.6 ft. long, how tall is the baby giraffe?





$\frac{17.4}{76.6} = \frac{x}{25.5}$

$\frac{76.6x}{76.6} = \frac{443.7}{76.6}$

$\boxed{5.79 \text{ feet}}$

SM 2

Date:

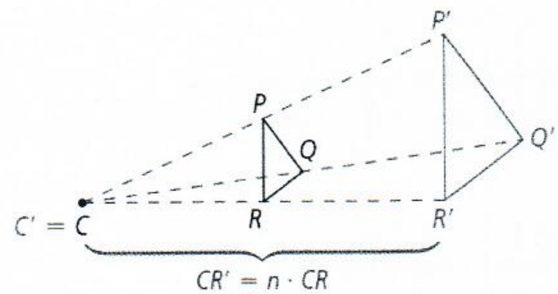
Section: 9.2

Objective: Dilations

Transformation: A change in the position, shape, or size of a geometric figure.

Examples of Transformations:

- ***reflections*** (flips)
- ***translations*** (slides)
- ***rotations*** (twists)
- ***dilations*** (enlargements or reductions)



Preimage: The original figure in a transformation.

Image: The resulting figure after the transformation.

Dilation: A transformation in which a larger or smaller copy of a figure is made that is similar to the original figure.

Enlargement: A dilation with a scale factor greater than 1. The image is larger than the preimage.

Reduction: A dilation with a scale factor between 0 and 1. The image is smaller than the preimage.

Properties of Dilations:

- If the scale factor is n, the segments in the image are n times as long as the corresponding segments in the preimage.
- The angles in the image are congruent to the corresponding angles in the preimage.
- The points on the image are n times as far away from the ***center of dilation*** as the points on the preimage.

Dilations with the Center at the Origin

If the center of dilation is the origin and the scale factor is n, the image of the point $A(x, y)$ will have coordinates $A'(nx, ny)$. In other words, multiply both the x and y coordinates by the scale factor to find the coordinates of the new point.

Examples: A dilation has center $(0, 0)$. Find the image of each point for the given scale factor.

a) $L(3, 0)$; scale factor = 5

$5(3) = 15$
$5(0) = 0$
$L'(15, 0)$

b) $N(-4, 7)$; scale factor = 0.2

$0.2(-4) = -0.8$
$0.2(7) = 1.4$
$N'(-0.8, 1.4)$

c) $A(6, 2)$; scale factor = 1.5

$1.5(6) = 9$
$1.5(2) = 3$
$A'(9, 3)$

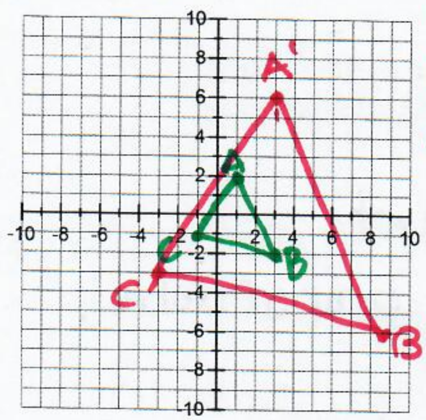
d) $F(3, -2)$; scale factor = $\frac{1}{3}$

$\frac{1}{3}(3) = 1$
$\frac{1}{3}(-2) = -2/3$
$F'(1, -2/3)$

Examples: Graph and label the figure with the given vertices. Then dilate the figure by the given scale factor with center $(0,0)$. Give the coordinates of the new vertices and graph the image.

a) $A(1,2)$, $B(3,-2)$, $C(-1,-1)$

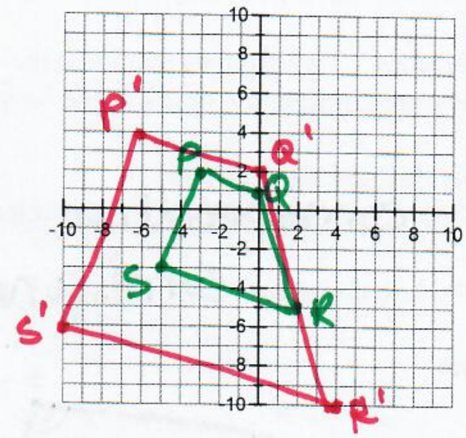
scale factor = 3



A'(3,6)
B'(9,-6)
C'(-3,-3)

b) $P(-3,2)$, $Q(0,1)$, $R(2,-5)$, $S(-5,-3)$

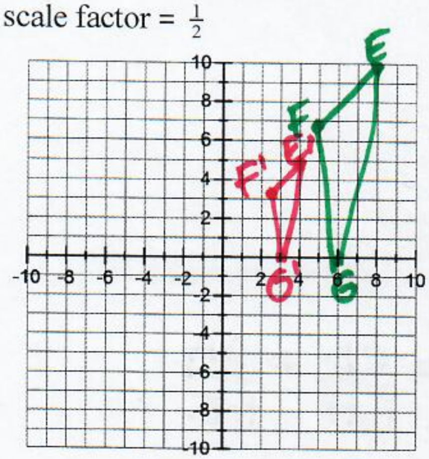
scale factor = 2



P'(-6,4)
Q'(0,2)
R'(4,-10)
S'(-10,-6)

c) $E(8,10)$, $F(5,7)$, $G(6,0)$

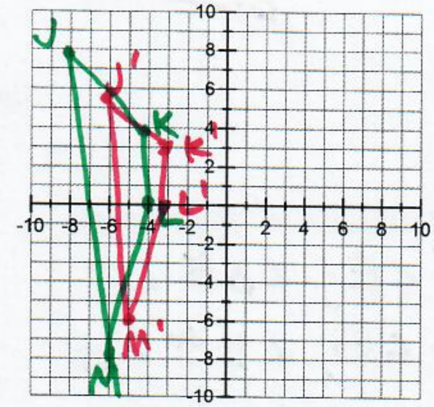
scale factor = $\frac{1}{2}$



E'(4,5)
F'(2.5,3.5)
G'(3,0)

d) $J(-8,8)$, $K(-4,4)$, $L(-4,0)$, $M(-6,-8)$

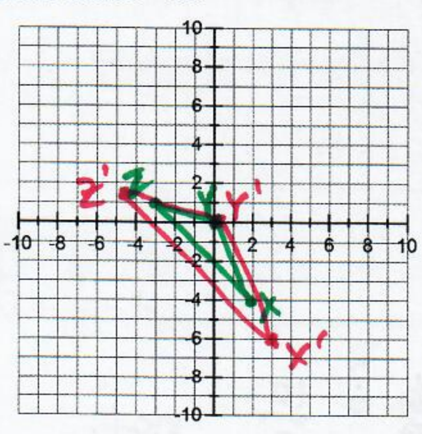
scale factor = 0.75



J'(-6,6)
K'(-3,3)
L'(-3,0)
M'(-4.5,-6)

e) $X(2,-4)$, $Y(0,0)$, $Z(-3,1)$

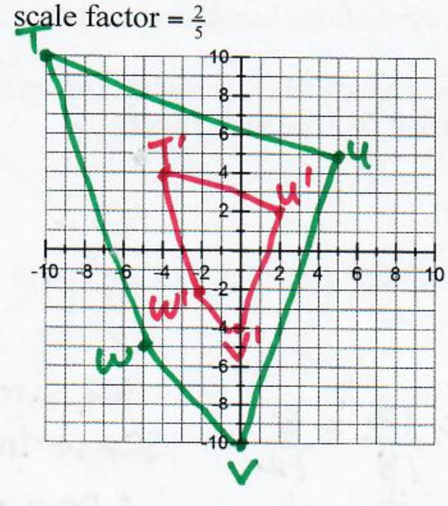
scale factor = 1.5



X'(3,-6)
Y'(0,0)
Z'(-4.5,1.5)

f) $T(-10,10)$, $U(5,5)$, $V(0,-10)$, $W(-5,-5)$

scale factor = $\frac{2}{5}$



T'(-4,4)
U'(2,2)
V'(0,-4)
W'(-2,-2)

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Date:

Section: 9.3

Objective: Similarity

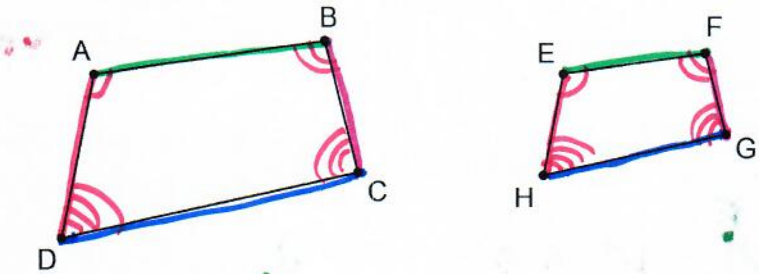
Congruent Figures: Two polygons are congruent if they are the same size and shape - that is, if their corresponding **angles** and sides are equal.

Similar Figures: Two **figures** that have the same shape are said to be **similar**. When two **figures** are **similar**, the ratios of the lengths of their corresponding sides are equal.

If two polygons are similar, then:

- Their corresponding **angles are** congruent.
- The lengths of their corresponding **sides are** proportional.

Examples:



Similarity Statement:
ABCD ~ EFGH

1. List all pairs of congruent angles.

$\angle A \cong \angle E$, $\angle B \cong \angle F$
$\angle C \cong \angle G$, $\angle D \cong \angle H$

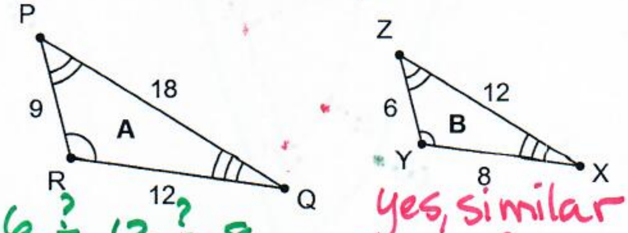
2. Write a ***statement of proportionality*** for the sides.

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

Scale Factor: In two similar geometric figures, the ratio of their corresponding sides is called the **scale factor**. To find the **scale factor**, locate two corresponding sides, one on each figure. Write the ratio of one length to the other to find the **scale factor** from one figure to the other.

Examples: Decide whether each set of figures are similar. If they are similar, write a similarity statement and find the scale factor.

1.

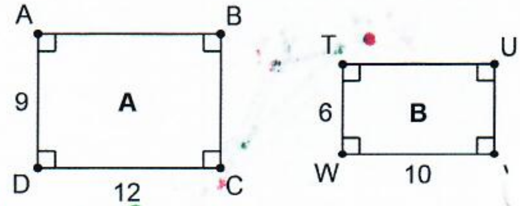


$\frac{6}{9} \stackrel{?}{=} \frac{12}{18} \stackrel{?}{=} \frac{8}{12}$

$\frac{2}{3}$ $\frac{2}{3}$ $\frac{2}{3}$

yes, similar
scale factor = 2/3
$\triangle PQR \sim \triangle ZXY$

2.

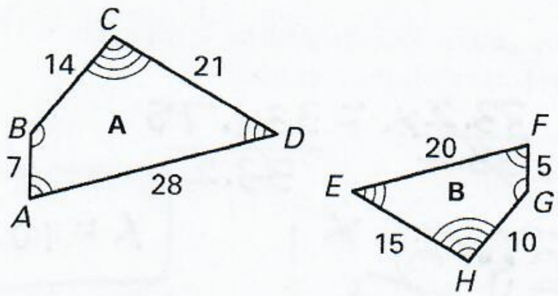


$\frac{6}{9} \stackrel{?}{=} \frac{10}{12}$

$\frac{2}{3}$ $\frac{5}{6}$

not similar

3.



$\frac{5}{7} \overset{?}{=} \frac{20}{28} \overset{?}{=} \frac{15}{21} \overset{?}{=} \frac{10}{14}$

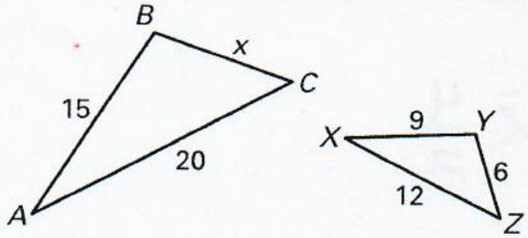
$\frac{5}{7} \quad \frac{5}{7} \quad \frac{5}{7} \quad \frac{5}{7}$

yes, similar
Scale factor = 5/7

CDAB ~ HEFG

Examples: $\triangle ABC \sim \triangle XYZ$. Find the value of x.

1.

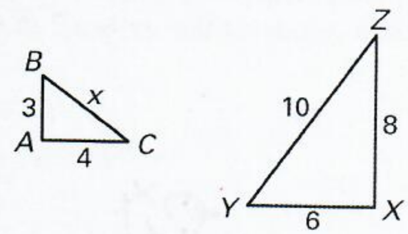


$\frac{15}{9} = \frac{x}{6}$

$\frac{9x}{9} = \frac{90}{9}$

$\boxed{x = 10}$

2.

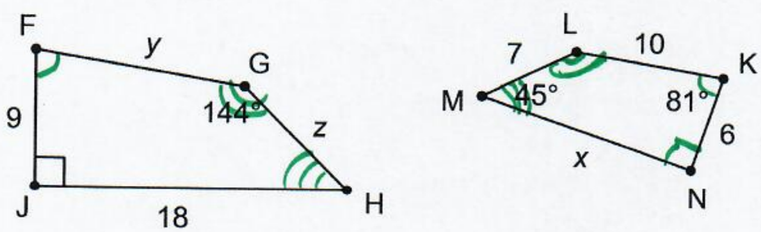


$\frac{3}{6} = \frac{x}{10}$

$\frac{30}{6} = \frac{6x}{6}$

$\boxed{5 = x}$

Examples: In the diagram below, $FGHJ \sim KLMN$.



1. List all pairs of congruent angles.

$\angle F \cong \angle K$, $\angle G \cong \angle L$, $\angle H \cong \angle M$, $\angle J \cong \angle N$

2. Write a statement of proportionality.

$\frac{FG}{KL} = \frac{GH}{LM} = \frac{HJ}{MN} = \frac{JF}{NK}$

3. Find $m\angle F$.

81°

4. Find $m\angle H$.

45°

5. Find $m\angle L$.

144°

6. Find $m\angle N$.

90°

7. Find the value of x.

$\frac{9}{6} = \frac{18}{x}$ $\boxed{x = 12}$

$\frac{9x}{9} = \frac{108}{9}$

8. Find the value of y.

$\frac{9}{6} = \frac{y}{10}$ $\boxed{y = 15}$

$\frac{90}{6} = \frac{6y}{6}$

9. Find the value of z.

$\frac{9}{6} = \frac{z}{7}$ $\boxed{z = 10.5}$

$\frac{63}{6} = \frac{6z}{6}$

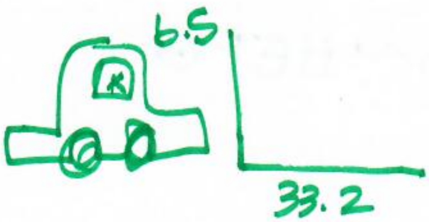
Examples:

1. A 6.5 ft. tall car standing next to an adult elephant casts a 33.2 ft. shadow. If the adult elephant casts a shadow that is 51.5 ft. long, then how tall is the elephant?

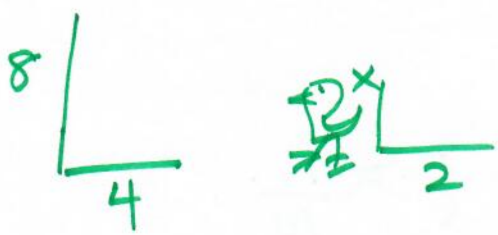
$$\frac{6.5}{x} = \frac{33.2}{51.5}$$

$$\frac{33.2x}{33.2} = \frac{334.75}{33.2}$$

$$\boxed{x = 10.08 \text{ ft}}$$



2. A telephone booth that is 8 ft. tall casts a shadow that is 4 ft. long. Find the height of a nearby lawn ornament that casts a 2 ft. shadow.



$$\frac{8}{x} = \frac{4}{2}$$

$$\frac{16}{4} = \frac{4x}{4}$$

$$\boxed{4 = x \text{ feet}}$$

SM 2

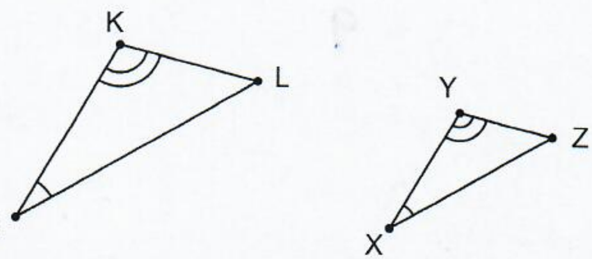
Date:

Section: 9.4

Objective: Triangle Similarity Theorems

We learned last time that to show two figures are similar, we've had to show that ***all*** of the corresponding angles are congruent and ***all*** of the corresponding sides are proportional. Luckily, there are some shortcuts for triangles.

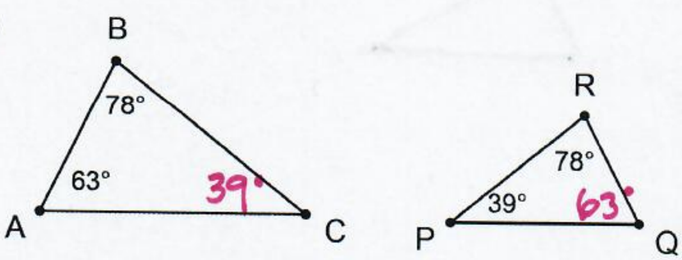
1. ***Angle-Angle Similarity Postulate (AA Similarity):***



If-then statement for the above triangles.
If $\angle J \cong \angle X$ and $\angle K \cong \angle Y$, then $\triangle JKL \sim \triangle XYZ$.

Examples: Determine whether the triangles are similar. **Explain** your reasoning. If they are similar, write a similarity statement.

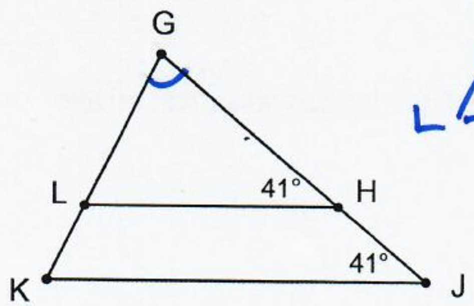
a)

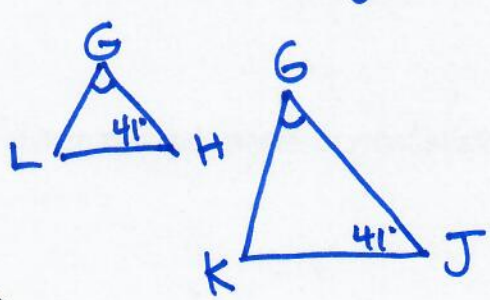


Two sets of congruent angles.

$\triangle ABC \sim \triangle QRP$ by AA similarity

b)

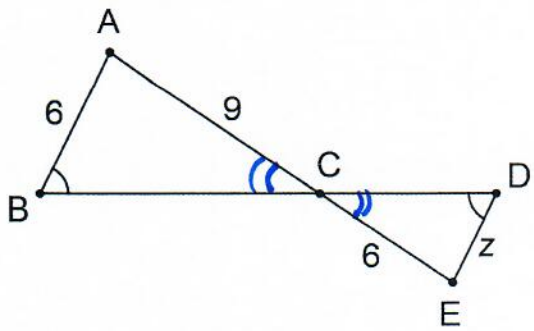




$\angle G$ is the same in both triangles, so there are two sets of congruent angles.

$\triangle GHL \sim \triangle GJK$ by AA similarity.

Example: Use the diagram to fill in the statements.



a) $\angle B \cong$ $\angle D$

b) $\angle ACB \cong$ $\angle ECD$ because they are vertical angles.

c) $\Delta ACB \sim$ ΔECD by the AA similarity postulate.

d) What is the scale factor? $\frac{6}{9} = \frac{2}{3}$

e) $\frac{AB}{DE} = \frac{AC}{EC}$

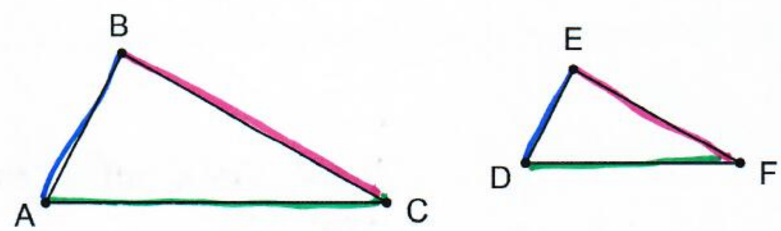
f) $\frac{6}{z} = \frac{9}{6}$

g) $z = ?$

$\frac{9z}{9} = \frac{36}{9}$

$\boxed{z = 4}$

2. ***Side-Side-Side Similarity Theorem (SSS Similarity):***

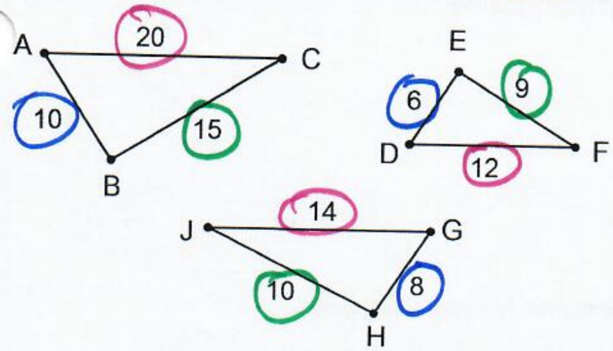


Write if-then statement for the above triangles.

If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then $\triangle ABC \sim \triangle DEF$.

★ **TIP:** When testing for SSS similarity, compare the shortest sides, longest sides, and middle length sides.

Example: Is either ΔDEF or ΔGHJ similar to ΔABC?



$$\frac{10}{6} \stackrel{?}{=} \frac{15}{9} \stackrel{?}{=} \frac{20}{12}$$

$$\frac{5}{3} \quad \frac{5}{3} \quad \frac{5}{3}$$

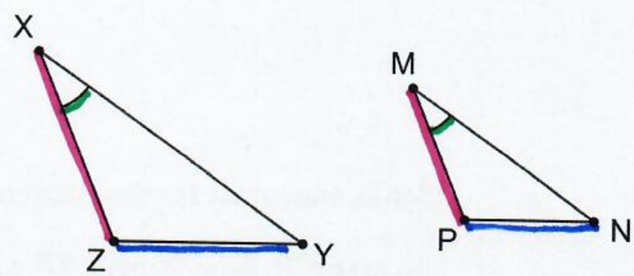
yes, ΔDEF is similar to ΔABC

$$\frac{10}{8} \stackrel{?}{=} \frac{15}{10} \stackrel{?}{=} \frac{20}{14}$$

$$\frac{5}{4} \neq \frac{3}{2} \neq \frac{10}{7}$$

no, ΔGHJ is not similar to ΔABC.

3. ***Side-Angle-Side Similarity Theorem (SAS Similarity):***

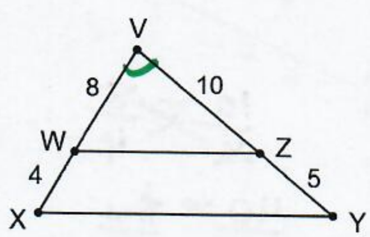


Write if-then statement for the above triangles.

If $\angle X \cong \angle M$ and $\frac{PM}{ZX} = \frac{MN}{XY}$, then $\Delta XYZ \sim \Delta MNP$.

Examples: Determine whether the triangles are similar. If they are similar, write a similarity statement and determine the scale factor.

a)



$$\frac{8}{12} \stackrel{?}{=} \frac{10}{15}$$

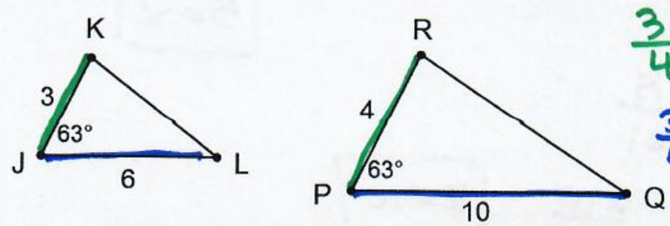
$$\frac{2}{3} = \frac{2}{3}$$

yes, by SAS

$\Delta WVZ \sim \Delta XVY$

Scale factor: $\frac{15}{10} = \frac{3}{2}$

b)



$$\frac{3}{4} \stackrel{?}{=} \frac{6}{10}$$

$$\frac{3}{4} \neq \frac{3}{5}$$

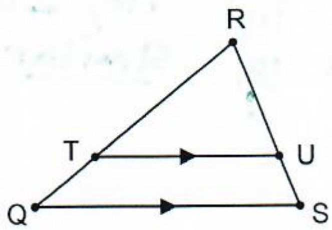
no, the triangles are not similar.

SM 2

Date: **Section: 9.5**

Objective: Triangle Proportionality and Midsegments

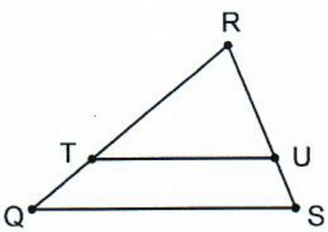
Triangle Proportionality Theorem:



If-then statement for the triangles.

In ΔQRS, if $\overline{TU} \parallel \overline{QS}$ then $\frac{RT}{TQ} = \frac{RU}{US}$.

Converse of the Triangle Proportionality Theorem:

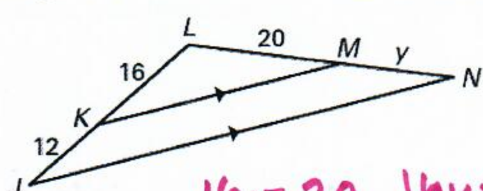


If-then statement for the triangles.

In ΔQRS, if $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

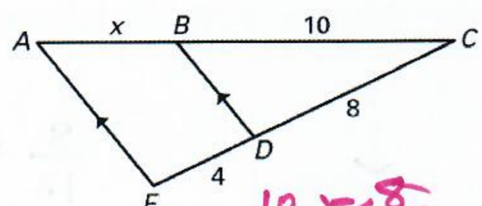
Examples: Find the value of the variable.

a)



$\frac{16}{12} = \frac{20}{y}$ $\frac{16y}{16} = \frac{240}{16}$ $\boxed{y=15}$

b)

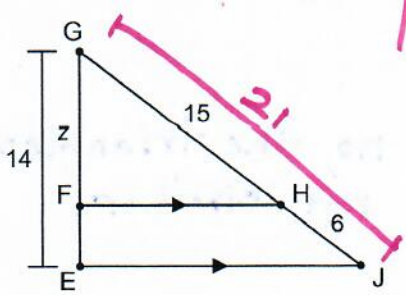


$\frac{10}{x} = \frac{8}{4}$

$\frac{40}{8} = \frac{8x}{8}$

$\boxed{5=x}$

c)

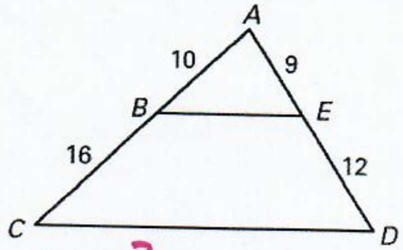


$\frac{z}{14} = \frac{15}{21}$

$\frac{21z}{21} = \frac{210}{21}$ $\boxed{z=10}$

Examples: Given the diagram, determine whether $\overline{BE} \parallel \overline{CD}$. Show work to support your answer.

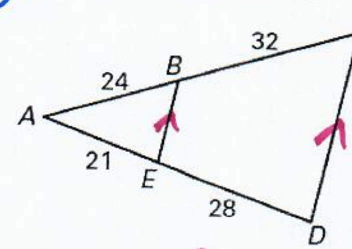
a)



$\frac{10}{16} \overset{?}{\neq} \frac{9}{12}$ $120 \neq 144$

$\frac{5}{8} \neq \frac{3}{4}$ not parallel.

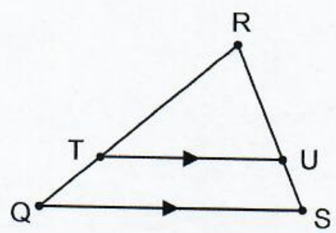
b)



$\frac{24}{32} \overset{?}{\neq} \frac{21}{28}$ $672 = 672$

$\frac{3}{4} \neq \frac{3}{4}$ parallel

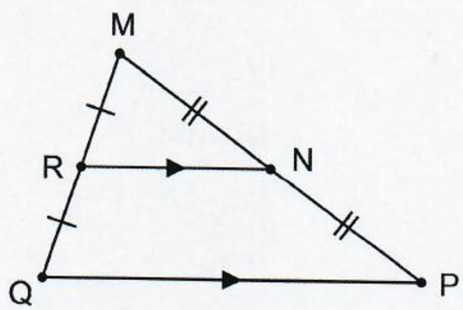
Example: Complete the proportion using the figure.



$\frac{QT}{QR} \cong \frac{SU}{SR}$

Midsegment of a Triangle: A segment that connects the midpoints of two sides of a triangle.

Midsegment Theorem: The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.



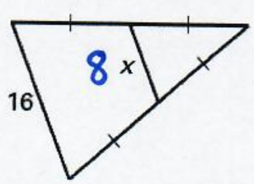
If-then statement for the triangles.

In $\triangle MPQ$, if $MR = RQ$ and $MN = NP$, then $\overline{RN} \parallel \overline{QP}$ and $RN = \frac{1}{2}QP$.

$QP = 2RN$

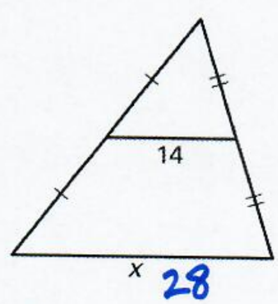
Examples: Find the value of the variable.

a)



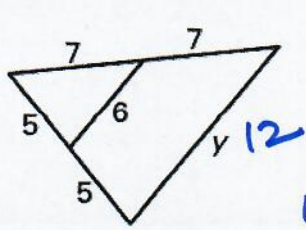
$x = 8$

b)



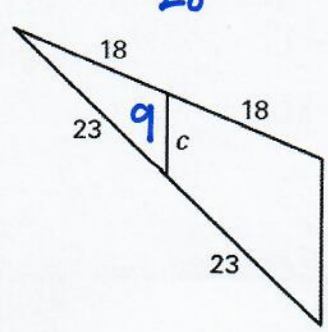
$x = 28$

c)



$y = 12$

d)



$c = 9$