SM 2

Date:

Section: 5.2 notes

Objective: Greatest Common Factor and Factor by Grouping

Factoring: The reverse of multiplying. It means figuring out what you would multiply together to get a polynomial, and writing the polynomial as the product of several factors (writing it as a multiplication problem).

Greatest Common Factor (GCF): The monomial with the largest possible coefficient and the variables with the largest possible exponents that divides evenly into every term of the polynomial.

Prime Polynomial: A polynomial that cannot be factored.

Factoring Out a Common Factor:

1. Find the GCF.
2. Use the distributive property in reverse to "factor out" the GCF:
   Write the GCF outside a set of parentheses.
   Inside the parentheses, write what is left when you *divide* the original terms by the GCF.

Note: If the GCF is the same as one of the terms of the polynomial, there will be a 1 left inside the parentheses.

3. If leading coefficient is negative, factor out a common factor with a negative coefficient.

Examples: Factor the following expressions.

a) $x^2 + 3x$

b) $-2y + 6$

c) $4n^2 - 20$

d) $15d^2 + 20d^4$

e) $2z^3 + 2z$

f) $-6h^2 + 3h$

g) $-20m^3 + 24m^2 - 32m$

h) $2a^2b^3c^4 + 8a^4b^8c^7 - 6a^3bc^5$

i) $p(q-6) + 2(q-6)$

Factoring by Grouping (4 Terms):

1. Factor out any common factors from all four terms first.
2. Look at the first two terms and the last two terms of the polynomial separately.
3. Factor out the GCF from the first two terms, write a plus sign (or a minus sign if the GCF on the last two terms is negative), then factor out the GCF from the last two terms.
4. You should have the same thing left in both sets of parentheses after you take out the GCFs. Factor out this common binomial factor from the two groups.

Examples: Factor the following expressions.

a) $x^3 - 4x^2 + 3x - 12$

b) $mp + mq + np + nq$

c) $4y^3 + 2y^2 - 6y - 3$

d) $20h^3 - 16h^2 - 5h + 4$

e) $4v^3 - 14v^2 + 12v - 42$

f) $4a - 7ab - 12 + 21b$

g) $6q^3 + 2q^2r - 36q - 12r$

h) $15w^3z^2 - 20w^2z - 60wz + 80$

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Date: **Section: 5.3 notes**

Objective: Factoring with leading coefficient other than 1

Review Examples: Multiply the following.

a) $(2x+3)(5x+4)$ b) $(3v-1)(v+2)$ c) $(4c-3)(7c-2)$

Factoring a Trinomial of the Form ax^2+bx+c by Grouping:

1. Always check for a GCF first! If there is a GCF, factor it out.
2. Multiply $\boldsymbol{a \cdot c}$.
3. Find two numbers that multiply to your answer $(\boldsymbol{a \cdot c})$ and add to ***b.***
4. Rewrite the middle term bx as **1st # $\cdot x$ + 2nd # $\cdot x$**
5. Factor the resulting polynomial by grouping.
6. If there are no numbers that multiply to $\boldsymbol{a \cdot c}$ and add to ***b,*** the polynomial is prime.

Examples: Factor the following polynomials using grouping.

a) $9h^2+9h+2$ b) $3x^2+19x+15$

c) $2z^2-11z+12$ d) $4p^2-20p+21$

e) $4n^2 - 20n + 25$

f) $10m^2 + 13m - 3$

g) $12y^2 + 30y - 72$

h) $8k^4 + 42k^3 - 36k^2$

i) $3r^2 - 16r - 12$

j) $4x^2 - 2xy - 12y^2$

k) $9x^2 - 4$

l) $100y^2 - 4$

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Date: **Section: 5.4 notes**

Objective: Review all Types of Factoring Problems

Step I. Factor Out the Greatest Common Factor:

1. Find the GCF.
2. Use the distributive property in reverse to "factor out" the GCF:
   Write the GCF outside a set of parentheses.
   Inside the parentheses, write what is left when you *divide* the original terms by the GCF.

Note: If the GCF is the same as one of the terms of the polynomial, there will be a 1 left inside the parentheses.

3. If leading coefficient is negative, factor out a common factor with a negative coefficient.

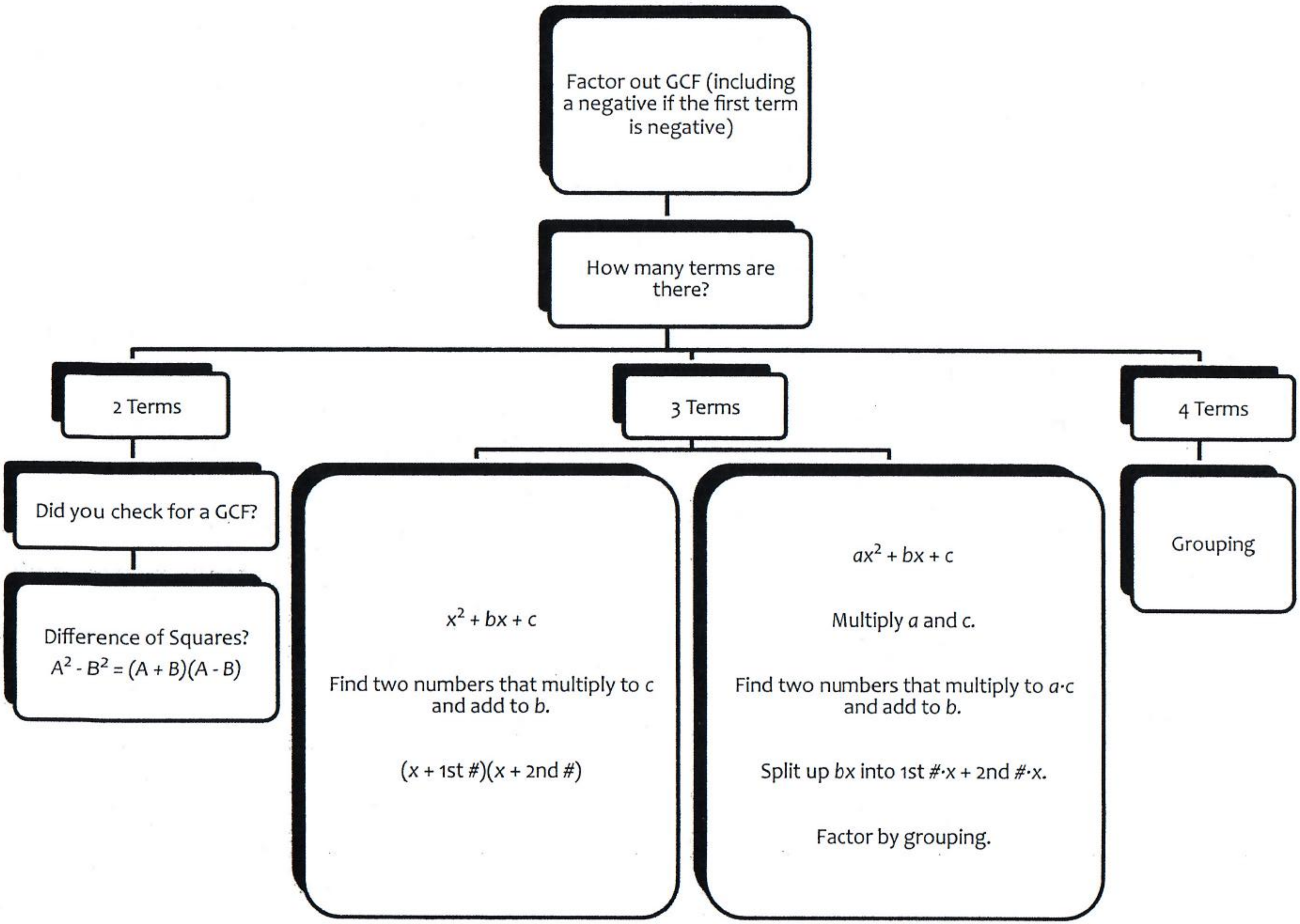
Step II. If the expression has 4 terms:
Factoring by Grouping (4 Terms):

1. Factor out any common factors from all four terms first.
2. Look at the first two terms and the last two terms of the polynomial separately.
3. Factor out the GCF from the first two terms, write a plus sign (or a minus sign if the GCF on the last two terms is negative), then factor out the GCF from the last two terms.
4. You should have the same thing left in both sets of parentheses after you take out the GCFs. Factor out this common binomial factor from the two groups.

Factoring a Trinomial of the Form $ax^2 + bx + c$ by Grouping:

1. Always check for a GCF first! If there is a GCF, factor it out.
2. Multiply $\boldsymbol{a \cdot c}$.
3. Find two numbers that multiply to your answer $(\boldsymbol{a \cdot c})$ and add to $\boldsymbol{b}$.
4. Rewrite the middle term $\boldsymbol{bx}$ as **1st** $\boldsymbol{\# \cdot x}$ **+ 2nd** $\boldsymbol{\# \cdot x}$
5. Factor the resulting polynomial by grouping.
6. If there are no numbers that multiply to $\boldsymbol{a \cdot c}$ and add to $\boldsymbol{b}$, the polynomial is prime.

Factoring Polynomials



SM2 5.4 Factoring (All Types) Notes

Factor each trinomial completely. Don't forget to check for a common factor first. If the polynomial is prime, say so.

1. $m^2 + 4m - 21$

2. $6r^2 - 5r - 4$

3. $x^2 - 49$

4. $6x^2 - 33x + 45$

5. $y^2 - 4y + 9$

6. $5v^2 - 45$

7. $12x^3 + 38x^2 + 20x$

8. $8m^2 + 2mn - 12mn - 3n^2$

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Date:

Section: 5.5A notes

Objective: Factoring with leading coefficient of 1
(no number in front of the first term)

Review Examples: Multiply the following.

a) $(x+3)(x+5)$

b) $(n-7)(n-4)$

c) $(t-2)(t+9)$

d) Look at your answers. How do the numbers in your answer relate to the numbers in the factors?

Factoring a Trinomial of the Form x^2+bx+c (the leading coefficient is 1):

1. **Always check for a GCF first!** If there is a GCF, factor it out.
2. Multiply a and c. Find the factors of ac.
3. Find the factors of ac that add to ***b*****.**
4. Rewrite the middle term bx as $\mathbf{1st\,\# \cdot x + 2nd\,\# \cdot x}$.
5. Factor the resulting polynomial by grouping.
6. If there are no numbers that multiply to c and add to b, the polynomial is prime.

Shortcut (This only works if there is no number in front of the first term.) The leading coefficient must be 1.

1. Find two numbers that multiply to c and add to b.
2. The factored form of x^2+bx+c is $(x+\text{1st \#})(x+\text{2nd \#})$.
3. The factored form of x^2-bx+c is $(x-\text{1st \#})(x-\text{2nd \#})$.
4. The factored form of x^2+bx-c *or* x^2-bx-c is $(x-\text{1st \#})(x+\text{2nd \#})$.
   The larger number will have the sign of the middle term.

Examples: Factor the following polynomials.

a) $x^2+11x+30$

b) $m^2+8m+12$

c) $2b^2+40b+144$

d) $q^2-15q+56$

e) $w^2 - 18w + 45$

f) $-5g^2 + 25g - 30$

g) $u^2 + 6u - 9$

h) $t^2 + 6t - 40$

i) $h^3 + h^2 - 12h$

j) $n^2 - 5n - 6$

k) $x^2 - 3x - 10$

l) $3x^2 - 6x + 15$

m) $x^2 - 4$

o) $3x^2 - 27$

p) $x^2 + 144$

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Date:

Section: 5.5B notes

Objective: Difference of two perfect squares

Review Examples: Multiply the following:

a) $(a+4)(a-4)$

b) $(3-k)(3+k)$

c) $(2m+7)(2m-7)$

Factoring a Difference of Squares:

- A polynomial of the form $A^2 - B^2$ is called a ***difference of squares.***
- Differences of squares always factor as follows: $A^2 - B^2 = (A+B)(A-B)$

★ This only works if ***both terms are perfect squares*** **<u>and</u>** ***you are subtracting.***

★ Don't forget to check for a GCF first!

Examples: Factor the following polynomials.

a) $x^2 - 25$

b) $m^2 - 81$

c) $w^2 + 36$

d) $49 - n^2$

e) $4t^2 - 1$

f) $9z^2 - 16$

g) $64y^2 - 81x^2$

h) $144k^2 + 25$

i) $2a^2 - 242$

j) $3 - 75p^2$

k) $100q^4r^2 - 9$

l) $x^4 - 16$