SM 2

UNIT 2 NOTES

Date: **Section: 2.2**

Objective: Analyzing Functions: Maxima, Minima, Increasing, Decreasing, Constant

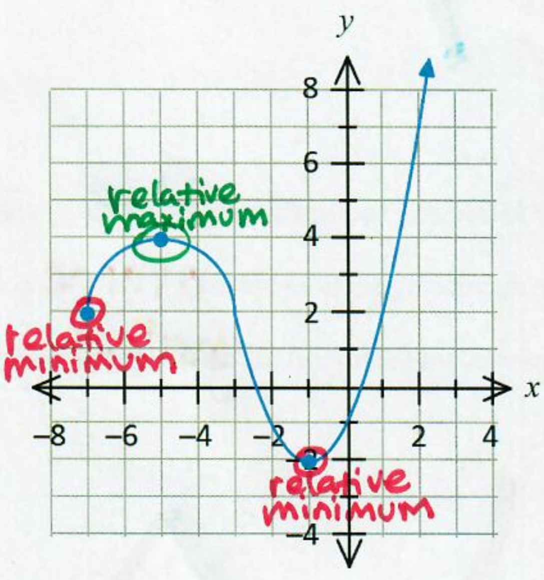
Relative Maxima and Minima

- When a point is higher than all the points near it, it is called a ***relative*** maximum.
- When a point is lower than all the points near it, it is called a ***relative*** minimum.
- If you are asked for a ***maximum or a minimum*** **point,** write the answer as an ordered pair.
- If you are asked for a ***maximum or a minimum*** **value,** the answer is the y-value.
- Infinity (positive or negative) is NOT a maximum or a minimum. ∞
- Maximum or minimum points are usually the endpoints or vertices.

Example:

a) Find the relative maximum point.

(-5, 4)



b) Find the relative maximum value.

4 (just the y-value)

c) Find the relative minimum points.

(-7, 2) (-1, -2)

d) Find the relative minimum values.

2, -2

Increasing, Decreasing, and Constant

If you look from left to right along the graph of the function, you will notice parts are *rising*, parts are *falling* and parts are *flat*. The different parts of the graph are described as intervals on which the function is *increasing*, *decreasing*, or *constant*, respectively.

- increasing: ***Uphill*** **from** ***left*** **to** ***right.***
- decreasing: ***Downhill*** **from** ***left*** **to** ***right.***
- Conctant: ***Flat.***

Writing Intervals Where the Graph is Increasing, Decreasing or Constant:

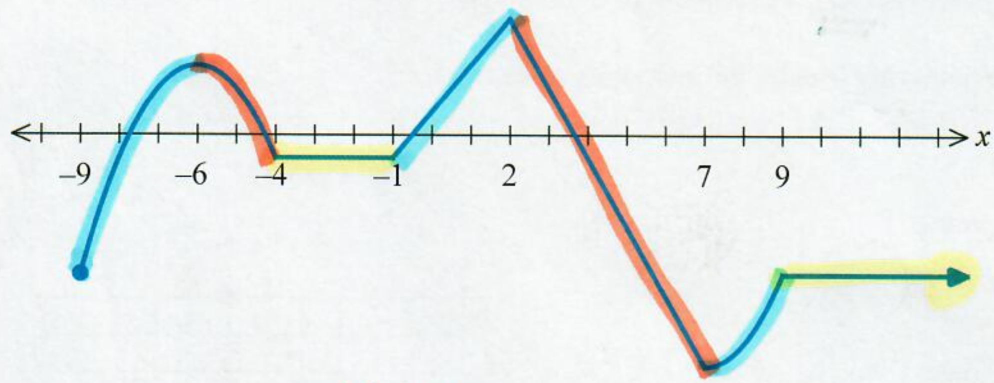
- Write the intervals of x -coordinates showing where the graph *starts* and *stops* going each direction from left to right.
- **Always use (and). Never use [and].**
- **Hint:** Look for places where the graph changes direction (relative maxima or relative minima) to help you break the graph into intervals.
- Use the ∪ sign to connect multiple intervals: (__,__)∪(__,__)

union

- **REMEMBER:** Only write down ***x*-coordinates! You might want to cross out the numbers on the *y*-axis to help you remember not to write down the *y*'s.**

Example: Highlight the increasing, decreasing, and constant sections of the graph each a different color. Then write the intervals where the graph is increasing, decreasing, and constant in interval notation.

a)



The increasing section(s) are blue color.

The decreasing section(s) are orange color.

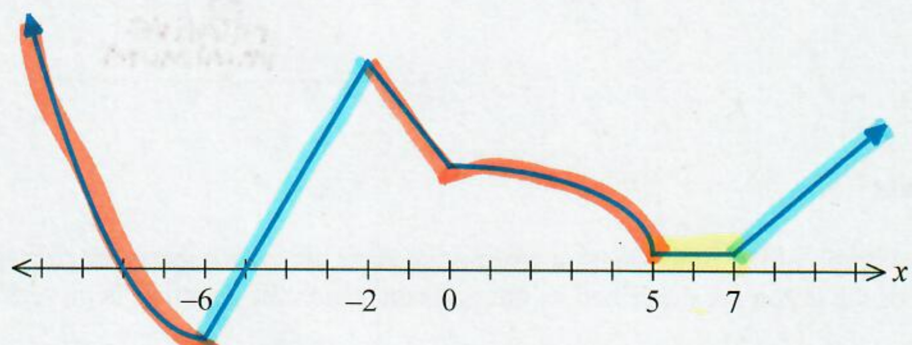
The constant section(s) are yellow color.

Increasing interval(s): (-9,-6)∪(-1,2)∪(7,9)

Decreasing interval(s): (-6,-4)∪(2,7)

Constant interval(s): (-4,-1)∪(9,∞)

b)



The increasing section(s) are blue color.

The decreasing section(s) are orange color.

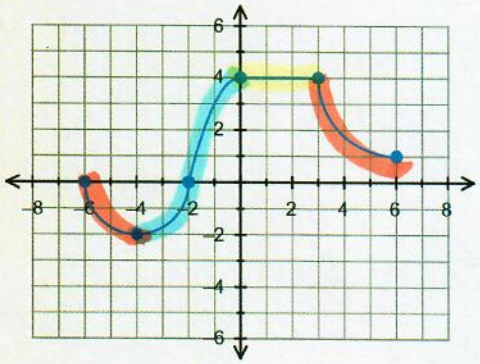
The constant section(s) are yellow color.

Increasing interval(s): (-6,-2)∪(7,∞)

Decreasing interval(s): (-∞,-6)∪(-2,5)

Constant interval(s): (5,7)

c)



Increasing:
Color: blue

Interval(s): $(-4, 0)$

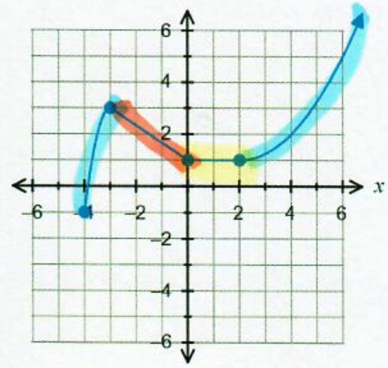
Decreasing:
Color: orange

Interval(s): $(-6, -4) \cup (3, 6)$

Constant:
Color: yellow

Interval(s): $(0, 3)$

d)



Increasing:
Color: blue

Interval(s): $(-4, -3) \cup (2, \infty)$

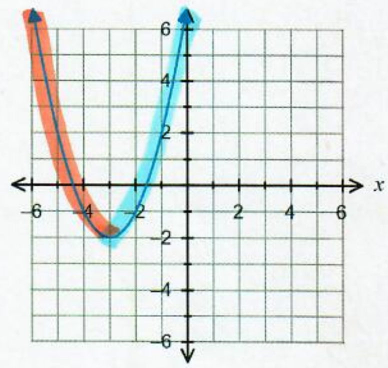
Decreasing:
Color: orange

Interval(s): $(-3, 0)$

Constant:
Color: yellow

Interval(s): $(0, 2)$

e)



Increasing:
Color: blue

Interval(s): $(-3, \infty)$

Decreasing:
Color: orange

Interval(s): $(-\infty, -3)$

Constant:
Color: n/a

Interval(s): n/a

SM 2

Date: **Section: 2.3**

Objective: Analyzing Functions: *x*- and *y*-intercepts, Positive and Negative

Intercepts

x-Intercepts: The points where a graph crosses the x-axis. They have the form $(x, 0)$.

- To find the *x*-intercept(s), plug in zero for y.

y-Intercepts: The points where a graph crosses the y axis. They have the form $(0, y)$.

- To find the *y*-intercept(s), plug in zero for x.

Examples: Find the intercepts of each graph. Write the intercepts as ordered pairs.

a) $f(x) = 2x + 6$

x-int: $0 = 2x + 6$, $-6 \quad -6$, $\frac{-6}{2} = \frac{2x}{2}$, $-3 = x$, $(-3, 0)$

y-int: $y = 2(0) + 6$, $y = 6$, $(0, 6)$

x-intercept $(-3, 0)$

y-intercept $(0, 6)$

b) $f(x) = -3x + 2$

x-int: $0 = -3x + 2$, $-2 \quad -2$, $\frac{-2}{-3} = \frac{-3x}{-3}$, $\frac{2}{3} = x$, $(\frac{2}{3}, 0)$

y-int: $y = -3(0) + 2$, $y = 2$, $(0, 2)$

***x*-intercept** $(2/3, 0)$

***y*-intercept** $(0, 2)$

c) $3x + 2y = 12$

x-int: $3x + 2(0) = 12$, $\frac{3x}{3} = \frac{12}{3}$, $x = 4$, $(4, 0)$

y-int: $3(0) + 2y = 12$, $\frac{2y}{2} = \frac{12}{2}$, $y = 6$ $(0, 6)$

x-intercept $(4, 0)$

y-intercept $(0, 6)$

d) $x - 2y = 5$

x-int: $x + 2(0) = 5$, $x = 5$, $(5, 0)$

y-int: $0 - 2y = 5$, $\frac{-2y}{-2} = \frac{5}{-2}$, $y = -5/2$ $(0, -5/2)$

***x*-intercept** $(5, 0)$

***y*-intercept** $(0, -5/2)$

e) $\frac{1}{2}x - 9y = 4$

x-int: $\frac{1}{2}x - 9(0) = 4$, $2 \cdot \frac{1}{2}x = 4 \cdot 2$, $x = 8$, $(8, 0)$

y-int: $\frac{1}{2}(0) - 9y = 4$, $\frac{-9y}{-9} = \frac{4}{-9}$, $y = -\frac{4}{9}$ $(0, -\frac{4}{9})$

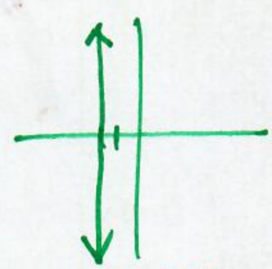
x-intercept $(8, 0)$

y-intercept $(0, -4/9)$

f) $-2x = 4$

x-int: $\frac{-2x}{-2} = \frac{4}{-2}$, $x = -2$, $(-2, 0)$

y-int: none



vertical line

***x*-intercept** $(-2, 0)$

***y*-intercept** none

Positive and Negative

- A function is ***positive*** where the *y*-coordinates are positive. The graph is above *the x-axis.*
- A function is ***negative*** where the *y*-coordinates are negative. The graph is below *the x-axis.*

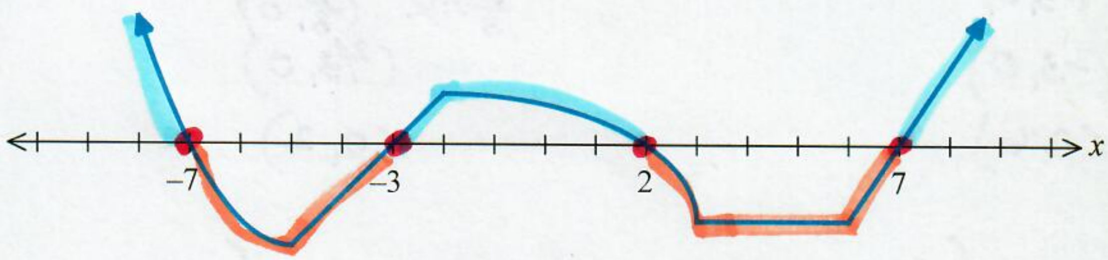
★ When you are asked to state where the graph is positive and negative, write the intervals of the of X - coordinates from left to right.

★ Use ( ) at the *x*-intercepts, where the graph crosses over from positive to negative. The *y*-coordinate is zero at the intercepts, so the graph is neither positive or negative there. That means those points are not included in the interval.

★ Use [ ] if the graph has an ***endpoint*** somewhere above or below the *x*-axis.

Example: Put a large dot on the *x*-intercepts. Highlight the positive and negative sections of the graph each a different color. Then write the intervals where the graph is positive or negative in interval notation.

a)

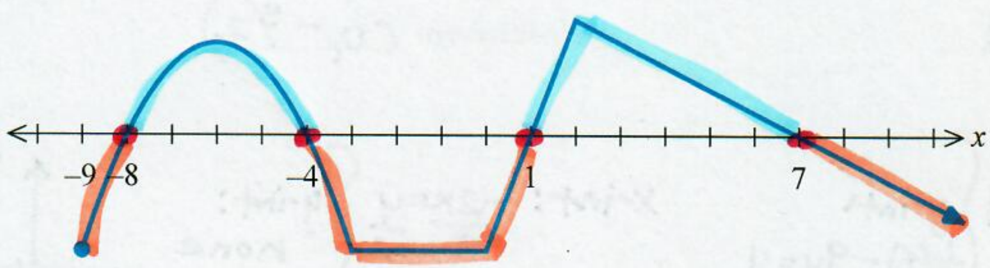


The *x*-intercepts are at -7, -3, 2, 7 ordered pairs: (-7,0), (-3,0), (2,0), (7,0)

The positive section(s) are blue color. Positive interval(s): $(-\infty,-7)\cup(-3,2)\cup(7,\infty)$

The negative section(s) are orange color. Negative interval(s): $(-7,-3)\cup(2,7)$

b)



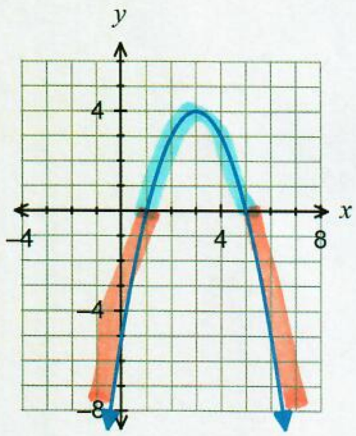
The *x*-intercepts are at -8, -4, 1, 7 ordered pairs: (-8,0), (-4,0), (1,0), (7,0)

The positive section(s) are blue color. Positive interval(s): $(-8,-4)\cup(1,7)$

The negative section(s) are orange color. Negative interval(s): $[-9,-8)\cup(-4,1)\cup(7,\infty)$

Example: Give the coordinates of the intercepts as ordered pairs. Then, highlight the parts of the graph where the function is positive and the parts where the function is negative in different colors. Write the intervals where the function is positive and negative in interval notation.

a)



x-intercept(s): (1,0), (5,0)

y-intercept: (0,-5)

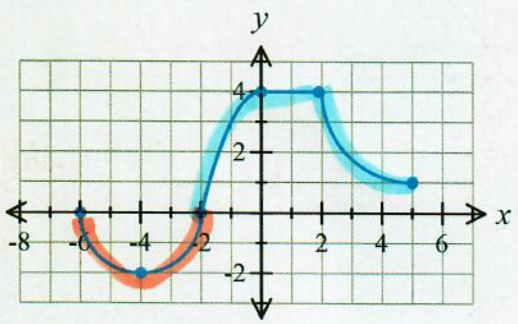
Positive color: blue

Positive interval(s): (1,5)

Negative color: orange

Negative interval(s): $(-\infty,1) \cup (5,\infty)$

b)



x-intercept(s): (-6,0), (-2,0)

y-intercept: (0,4)

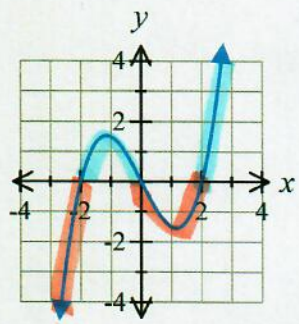
Positive color: blue

Positive interval(s): (-2, 5]

Negative color: orange

Negative interval(s): (-6,-2)

c)



x-intercept(s): (-2,0), (0,0), (2,0)

y-intercept: (0,0)

Positive color: blue

Positive interval(s): $(-2,0) \cup (2,\infty)$

Negative color: orange

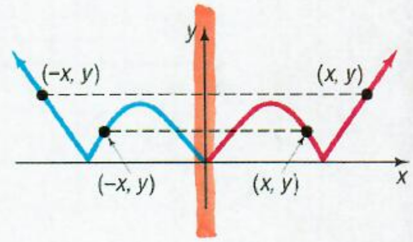
Negative interval(s): $(-\infty,-2) \cup (0,2)$

SM 2

Date: **Section: 2.4**

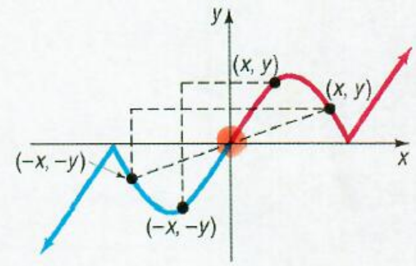
Objective: Analyzing Functions: Symmetry and End Behavior

Symmetry



y-axis or even ***symmetry:***

- The left and right sides are mirror images around the y -axis.
- The left and right sides would overlap if you fold the graph along the y -axis.

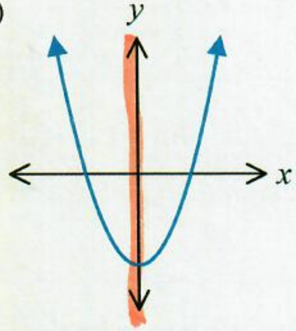


Origin or odd ***symmetry***

- When you rotate the graph around 180°, you end up with the same graph you started with.
- If you fold the graph along the x-axis and then again along the y-axis, the two halves would overlap.

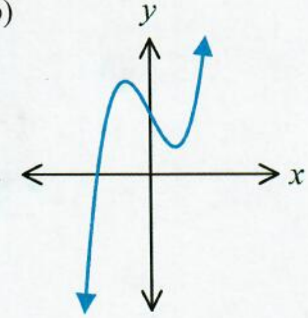
Examples: Determine what type of symmetry each function has (even, odd, or neither).

a)



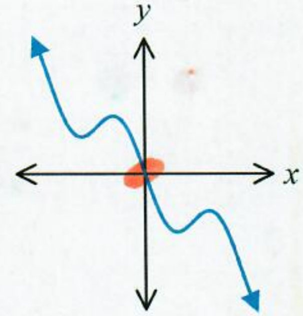
even

b)



neither

c)



odd

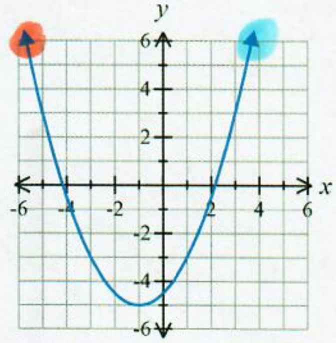
End Behavior

End behavior describes what is happening to the ***y*-coordinates** of the graph as you move left $(x \to -\infty)$ or as you move right $(x \to \infty)$.

- **Left end behavior** looks like this: $\lim_{x \to -\infty} f(x) =$ ___.
- **Right end behavior** looks like this: $\lim_{x \to \infty} f(x) =$ ___.
- **Arrow pointing up:** Write ∞
- **Arrow pointing down:** Write $-\infty$
- **Endpoint (no arrow):** Write D.N.E. (does not exist)
- **Asymptote or flat end with arrow:** Write y-coordinate of asymptote or flat part

Examples: Describe the end behavior of each graph using limits.

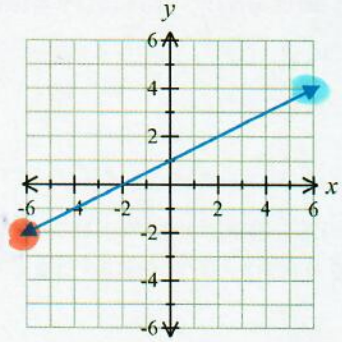
a)



Left: $\lim_{x\to-\infty} f(x) = \underline{\infty}$

Right: $\lim_{x\to\infty} f(x) = \underline{\infty}$

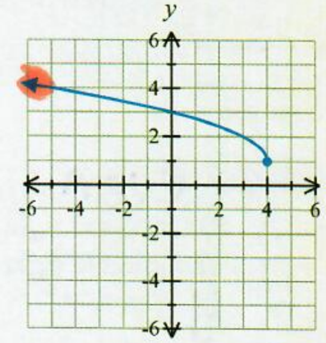
b)



Left: $\lim_{x\to-\infty} f(x) = \underline{-\infty}$

Right: $\lim_{x\to\infty} f(x) = \underline{\infty}$

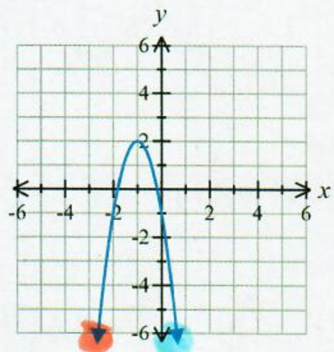
c)



Left: $\lim_{x\to-\infty} f(x) = \underline{\infty}$

Right: $\lim_{x\to\infty} f(x) = \underline{\text{DNE}}$

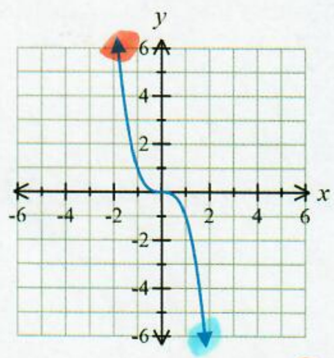
d)



Left: $\lim_{x\to-\infty} f(x) = \underline{-\infty}$

Right: $\lim_{x\to\infty} f(x) = \underline{-\infty}$

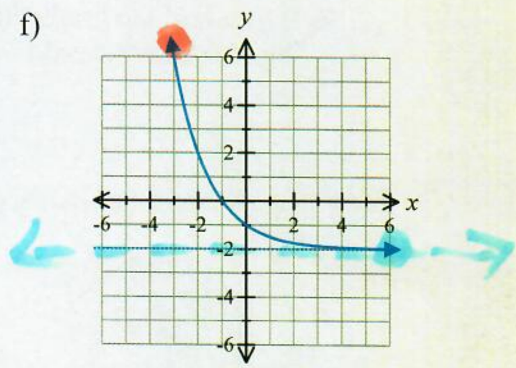
e)



Left: $\lim_{x\to-\infty} f(x) = \underline{\infty}$

Right: $\lim_{x\to\infty} f(x) = \underline{-\infty}$

f)

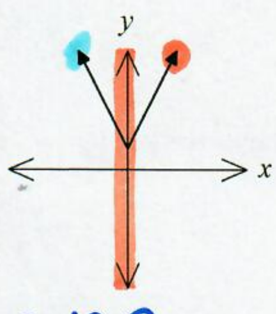


Left: $\lim_{x\to-\infty} f(x) = \underline{\infty}$

Right: $\lim_{x\to\infty} f(x) = \underline{-2}$

Determine if graph is even, odd, or neither. Then find right end behavior and left end behavior.

a.

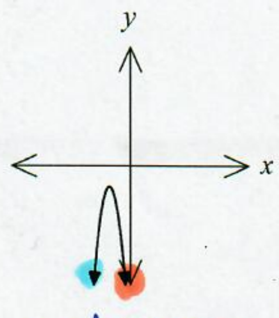


even

left: $\lim_{x\to-\infty} f(x) = \infty$

right: $\lim_{x\to\infty} f(x) = \infty$

b.

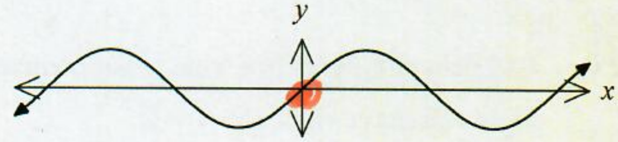


neither

left: $\lim_{x\to-\infty} f(x) = -\infty$

right: $\lim_{x\to\infty} f(x) = -\infty$

c.



odd

left: $\lim_{x\to-\infty} f(x) = \text{DNE}$

right: $\lim_{x\to\infty} f(x) = \text{DNE}$