SM 2 **Date:** **Section: 12.1**

Objective: Circle Vocabulary, Arc, and Angle Measures

Circle: All points in a plane that are the same distance from a given point, called the ***center*** of the circle.

Chord: A segment with both endpoints on a circle.

$\overline{AB}$ is a chord.

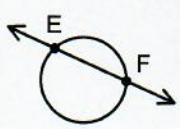
Diameter: A chord that passes through the center of a circle.

$\overline{MN}$ is a diameter.

Radius: A segment with one endpoint on the circle and one endpoint at the center of the circle.

$\overline{CD}$ is a radius.

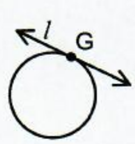
Secant: A line that intersects a circle at two points.



$\overleftrightarrow{EF}$ is a secant.

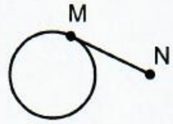
Tangent: A line in the plane of the circle that intersects a circle at exactly one point.

Point of Tangency: The point where a tangent intersects a circle.



Line l is a tangent. G is the point of tangency.

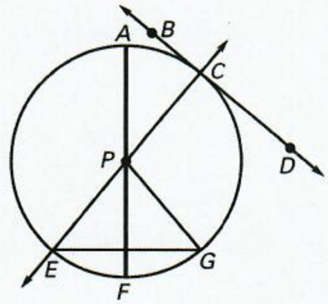
Tangent Segment: A segment that touches a circle at one of its endpoints and lies in the line that is tangent to the circle at that point.



$\overline{MN}$ is a tangent segment.

Example: In circle P, name the term that best describes the given line, segment, or point.

$\overline{AF}$	diameter	C	point of tangency
$\overline{EG}$	chord	$\overleftrightarrow{CE}$	secant
$\overline{PF}$	radius	$\overleftrightarrow{BD}$	tangent
$\overline{PG}$	radius	P	center



Example: In $\odot Q$, identify a chord, a diameter, two radii, a secant, two tangents, and two points of tangency.

Chord: $\overline{KM}$

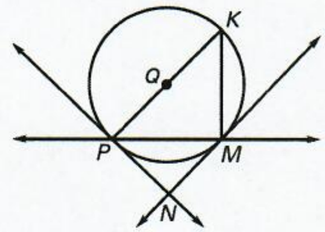
Diameter: $\overline{PK}$

Radii: $\overline{QP}$, $\overline{QK}$

Secant: $\overleftrightarrow{PM}$

Tangents: $\overleftrightarrow{PN}$, $\overleftrightarrow{MN}$

Points of tangency: P, M



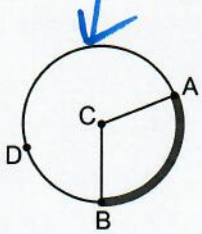
Central Angle: An angle in a circle whose vertex is the center of the circle and whose sides are radii of the circle

$\angle ACB$

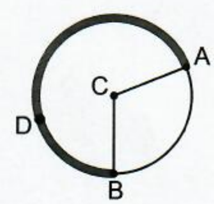
Minor Arc: All the points on a circle that lie in the interior of a central angle whose measure is less than 180°.

Major Arc: All the points on a circle that do not lie on the corresponding minor arc.

$\overset{\frown}{AB}$ is a minor arc.



$\overset{\frown}{ADB}$ is a major arc.

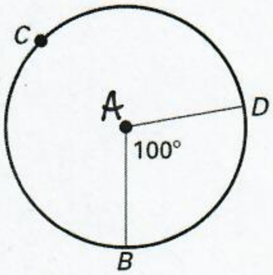


Measure of a Central Angle: is the measure of the angle with its vertex at the center of a circle.

Measure of a Minor Arc: is the measure of its central angle.

Measure of a Major Arc: 360° minus the measure of the minor arc.

Example:



Measure of central angle: 100°

Measure of the minor arc: 100°

Measure of the major arc: 260°

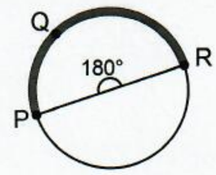
360 − 100°

Name the central angle: $\angle DAB$

Name the minor arc: $\overset{\frown}{DB}$

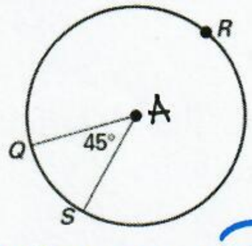
Name the major arc: $\overset{\frown}{DCB}$

Semicircle: An arc whose central angle measures 180°.



Examples: Name the major and minor arcs and the central angle. Find the measure of each.

a)

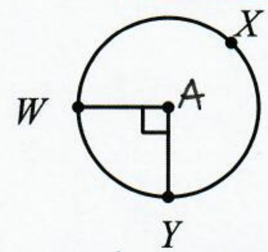


major arc: $\widehat{QRS}$ 315°

minor arc: $\widehat{QS}$ 45°

central angle: ∠QAS 45°

b)

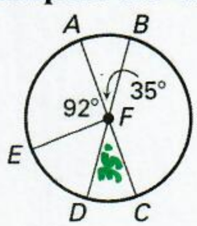


major arc: $\widehat{WXY}$ 270°

minor arc: $\widehat{WY}$ 90°

central angle: ∠WAY 90°

Examples: $\overline{AC}$ and $\overline{BD}$ are diameters. Find the indicated measures.



a) $m\widehat{DC}$ 35°

b) $m\widehat{BC}$ = 145°

180° − 35°

c) $m\widehat{CDE}$ = 88°

180° − 92°

d) $m\widehat{DE}$ = 53°

180° − 92° − 35°

e) $m\widehat{ABE}$ = 268°

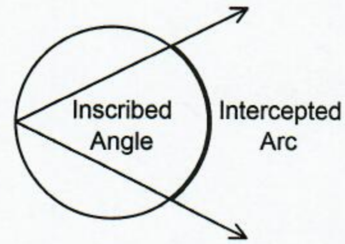
360° − 92°

f) $m\widehat{ABD}$ = 215°

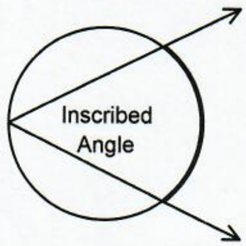
180° + 35°

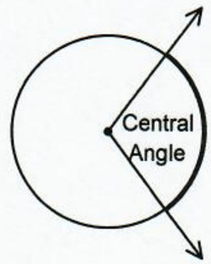
Inscribed Angle: An angle whose vertex is on a circle and whose sides contain chords of the circle.

Intercepted Arc: An arc that lies in the interior of an inscribed angle and has endpoints on the sides of the angle.

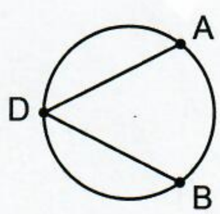


WARNING: Don't get ***inscribed*** angles and ***central*** angles mixed up!





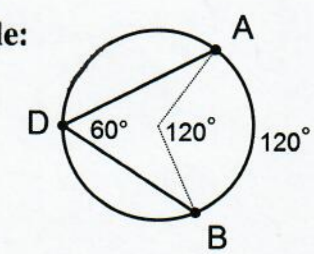
Theorem: If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.



$$m\angle ADB = \frac{1}{2} m\overparen{AB}$$

$$m\overparen{AB} = 2m\angle ADB$$

Example:

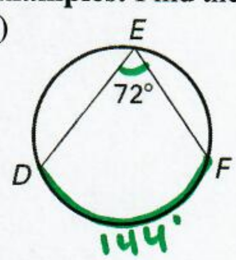


$$m\angle ADB = 60°$$

$$m\overparen{AB} = 120°$$

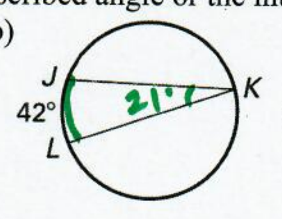
Examples: Find the measure of the inscribed angle or the intercepted arc.

a)



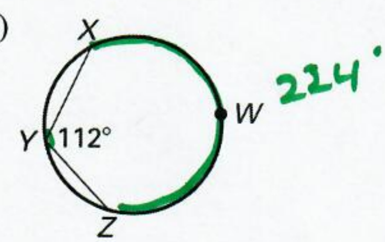
$2 \cdot 72° = 144°$

b)



$\frac{1}{2} \cdot 42° = 21°$

c)



$2 \cdot 112° = 224°$

SM 2

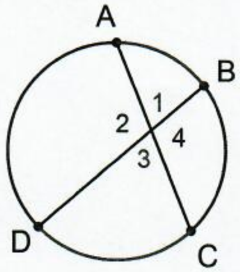
Date:

Section: 12.2

Objective: Tangent and Chord Theorems

Theorem:

- If two chords intersect inside a circle, then the measure of each angle formed is the average of the measures of the arcs intercepted by the angle and its vertical angle.

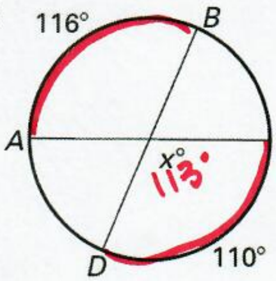


$$m\angle 1 = m\angle 3 = \frac{1}{2}\left(m\widehat{AB} + m\widehat{CD}\right) \quad \text{OR} \quad \frac{m\widehat{AB} + m\widehat{CD}}{2}$$

$$m\angle 2 = m\angle 4 = \frac{1}{2}\left(m\widehat{BC} + m\widehat{AD}\right) \quad \text{OR} \quad \frac{m\widehat{BC} + m\widehat{AD}}{2}$$

Examples: Find the value of x.

a)

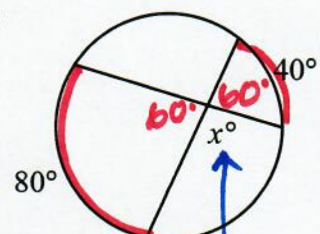


$$\frac{116^\circ + 110^\circ}{2}$$

$$\frac{226^\circ}{2}$$

$$\boxed{x = 113}$$

b)



$$180^\circ - 60^\circ$$

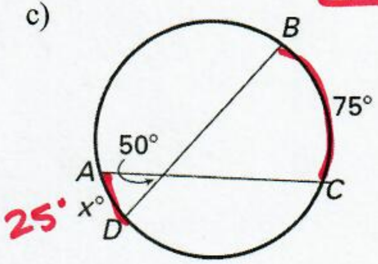
$$\boxed{x = 120}$$

$$\frac{40^\circ + 80^\circ}{2}$$

$$\frac{120^\circ}{2}$$

$$= 60^\circ$$

c)

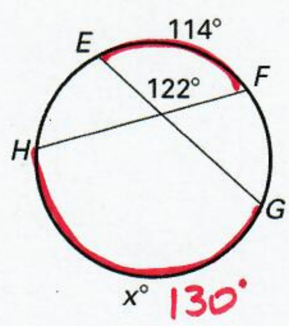


$$\cancel{2} \cdot \frac{x + 75}{\cancel{2}} = 50 \cdot 2$$

$$x + \cancel{75} = 100$$
$$-\cancel{75} \quad -75$$

$$\boxed{x = 25}$$

d)

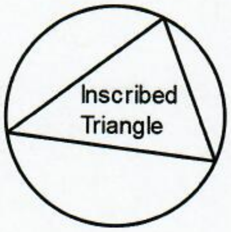


$$\cancel{2} \cdot \frac{x + 114}{\cancel{2}} = 122 \cdot 2$$

$$x + \cancel{114} = 244$$
$$-\cancel{114} \quad -114$$

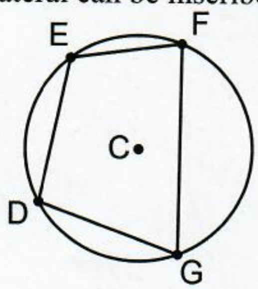
$$\boxed{x = 130}$$

Inscribed Polygon: A polygon whose vertices all lie on a circle.



Theorem:

- If a quadrilateral can be inscribed in a circle, then its opposite angles are supplementary.

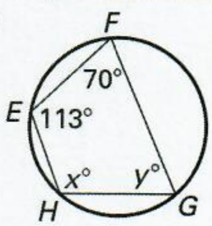


$m\angle D + m\angle F = 180°$

$m\angle E + m\angle G = 180°$

Examples: Find the values of x and y.

a)



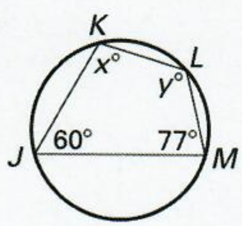
$x = 180 - 70$

$\boxed{x = 110}$

$y = 180 - 113$

$\boxed{y = 67}$

b)



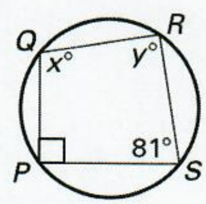
$x = 180 - 77$

$\boxed{x = 103}$

$y = 180 - 60$

$\boxed{y = 120}$

c)



$x = 180 - 81$

$\boxed{x = 99}$

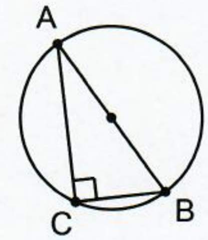
$y = 180 - 90$

$\boxed{y = 90}$

Theorems:

- If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.

  If $\triangle ABC$ is a right triangle with hypotenuse $\overline{AB}$, then $\overline{AB}$ is a diameter of the circle.

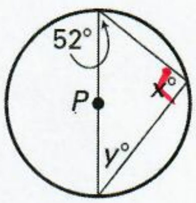


- If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle.

  If $\overline{AB}$ is a diameter of the circle, then $\triangle ABC$ is a right triangle with $\overline{AB}$ as hypotenuse.

Examples: Find the values of x and y in $\odot$P.

a)

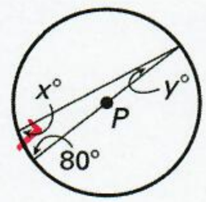


$x=90$

$y=90-52$

$\boxed{y=38}$

b)

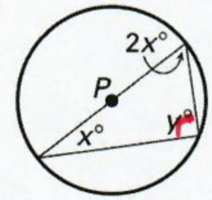


$x=90$

$y=90-80$

$\boxed{y=10}$

c)



$y=90$

$x+2x=90$

$\frac{3x}{3}=\frac{90}{3}$

$\boxed{x=30}$

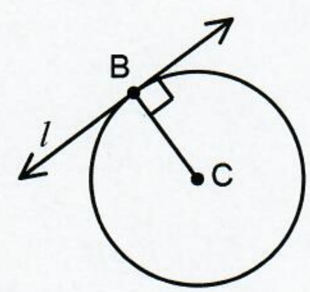
Theorems About Tangents:

- If a line is tangent to a circle, then it is perpendicular to the radius drawn at the point of tangency.

  If line l is tangent to $\odot C$ at B, then $l \perp \overline{CB}$.

- In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

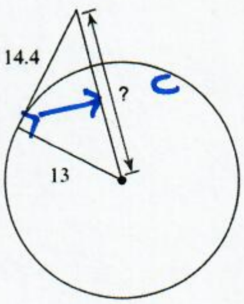
  If $l \perp \overline{CB}$, then line l is tangent to $\odot C$ at B.



$a^2+b^2=c^2$

Examples: Find the length of the missing segment. Assume that segments which appear to be tangent to the circle are tangent to the circle.

a)



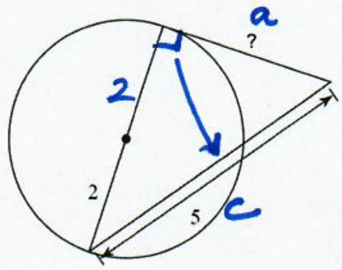
$13^2+14.4^2=c^2$

$169+207.36=c^2$

$\sqrt{376.36}=\sqrt{c^2}$

$\boxed{19.4=c}$

b)



$a^2+4^2=5^2$

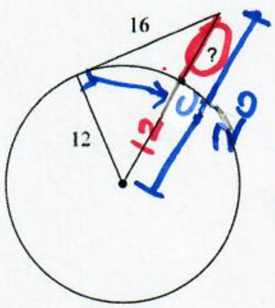
$a^2+16=25$

$-16\quad -16$

$\sqrt{a^2}=\sqrt{9}$

$\boxed{a=3}$

c)



$12^2+16^2=c^2$

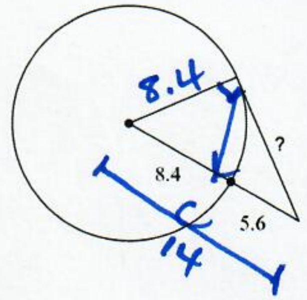
$144+256=c^2$

$\sqrt{400}=\sqrt{c^2}$

$20=c$

$?=20-12=\boxed{8}$

d)



$8.4^2+b^2=14^2$

$70.56+b^2=196$

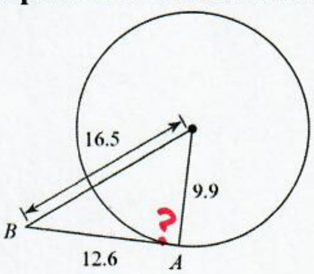
$-70.56\quad -70.56$

$\sqrt{b^2}=\sqrt{125.44}$

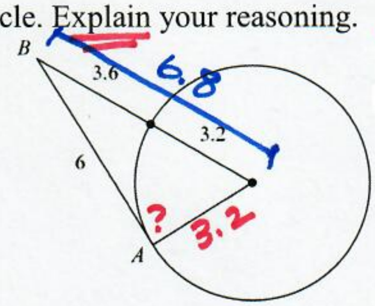
$\boxed{b=11.2}$

Examples: Determine whether $\overline{AB}$ is tangent to the circle. Explain your reasoning. * Does $a^2+b^2=c^2$?

a)



b)



$12.6^2+9.9^2 \stackrel{?}{=} 16.5^2$

$158.76+98.01 \stackrel{?}{=} 272.25$

$256.77 \neq 272.25$

$\overline{AB}$ is <u>not</u> tangent to the circle because $a^2+b^2 \neq c^2$.

$3.2^2+6^2 \stackrel{?}{=} 6.8^2$

$10.24+36 \stackrel{?}{=} 46.24$

$46.24 = 46.24$

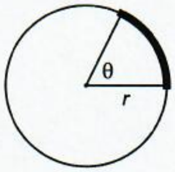
yes, $\overline{AB}$ is tangent to the circle because $a^2+b^2=c^2$.

SM 2

Date: **Section: 12.3**

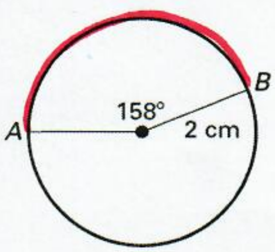
Objective: Arc Length and Sector Area

Arc Length: $Arc\text{ Length} = \frac{\theta}{360°} \cdot \text{circumference of circle} = \frac{\theta}{360°} \cdot 2\pi r$

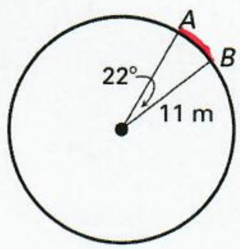


Examples: Find the length of $\overset{\frown}{AB}$. Write your answers in terms of π and as decimals rounded to the nearest hundredth.

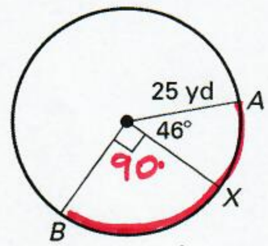
a)



b)



c)



$\frac{158}{360} \cdot 2\pi(2)$

$= \frac{79\pi}{45} \approx 5.52\text{ cm}$

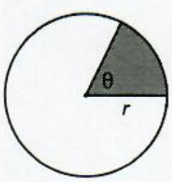
$\frac{22}{360} \cdot 2\pi(11)$

$= \frac{121\pi}{360} \approx 4.22\text{ m}$

$\frac{136}{360} \cdot 2\pi(25)$

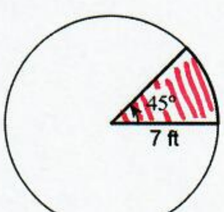
$= \frac{179\pi}{9} \approx 59.34\text{ yd}$

Sector Area: $\text{Sector Area} = \frac{\theta}{360°} \cdot \text{area of circle} = \frac{\theta}{360°} \cdot \pi r^2$

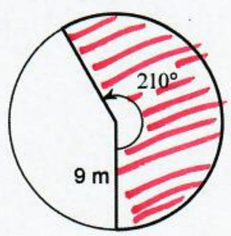


Examples: Find the area of each sector. Write your answers in terms of π and as decimals rounded to the nearest tenth.

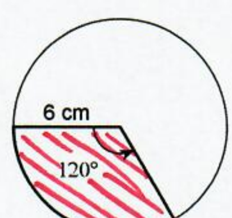
a)



b)



c)



$\frac{45}{360} \cdot \pi(7)^2$

$= \frac{49\pi}{8} \approx 19.2\text{ ft}^2$

$\frac{210}{360} \cdot \pi(9)^2$

$= \frac{189\pi}{4} \approx 148.4\text{ m}^2$

$\frac{120}{360} \cdot \pi(6)^2$

$= 12\pi \approx 37.7\text{ cm}^2$

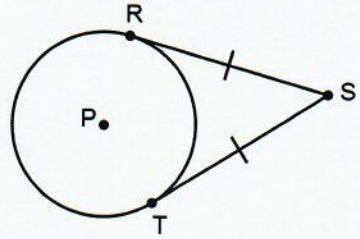
SM 2

Date: **Section: 12.4**

Objective: More Tangent and Chord Theorems

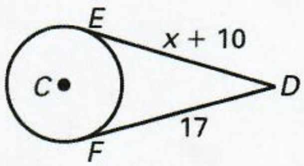
Theorem: If two segments from the same point outside a circle are both tangent to the circle, then they are congruent.

If $\overline{SR}$ and $\overline{ST}$ are tangent to circle P at points R and T then $\overline{SR} \cong \overline{ST}$.



Examples: $\overline{DE}$ and $\overline{DF}$ are both tangent to ⊙ C. Find the value of x.

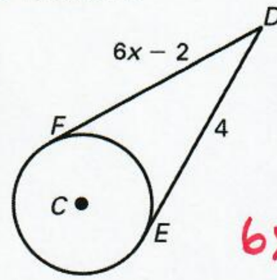
a)



$x + \cancel{10} = 17$
$-\cancel{10} \quad -10$

$\boxed{x = 7}$

b)

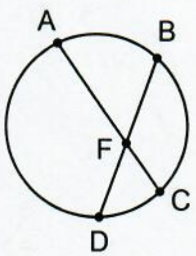


$6x - \cancel{2} = 4$
$+\cancel{2} \quad +2$

$\frac{\cancel{6}x}{\cancel{6}} = \frac{6}{6}$

$\boxed{x = 1}$

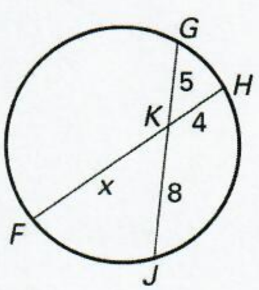
Theorem: If two chords intersect inside a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.



$\overline{AF} \cdot \overline{FC} = \overline{BF} \cdot \overline{FD}$

Examples: Find the value of x.

a)

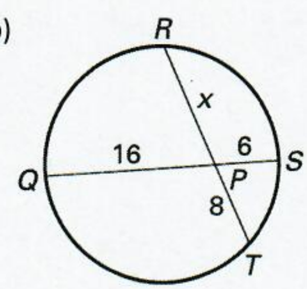


$5 \cdot 8 = 4 \cdot x$

$\frac{40}{4} = \frac{\cancel{4}x}{\cancel{4}}$

$\boxed{10 = x}$

b)

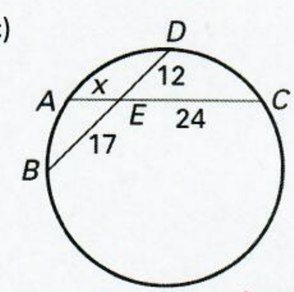


$16 \cdot 6 = 8 \cdot x$

$\frac{128}{8} = \frac{\cancel{8}x}{\cancel{8}}$

$\boxed{16 = x}$

c)



$17 \cdot 12 = 24 \cdot x$

$\frac{204}{24} = \frac{\cancel{24}x}{\cancel{24}}$

$\boxed{8.5 = x}$

SM 2

Date:

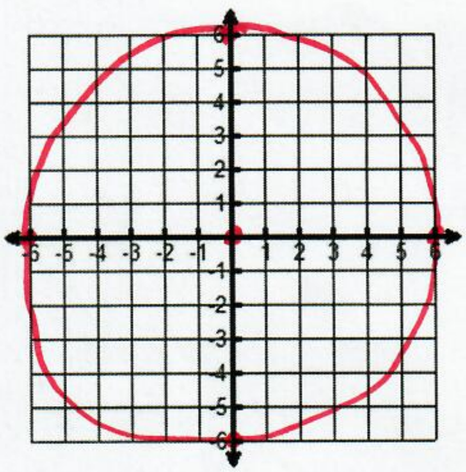
Section: 12.5

Objective: Graphing Circles

Equation of a Circle with Center at the Origin and Radius r: $x^2+y^2=r^2$

Examples: Determine the center and radius of each circle, then graph the circle.

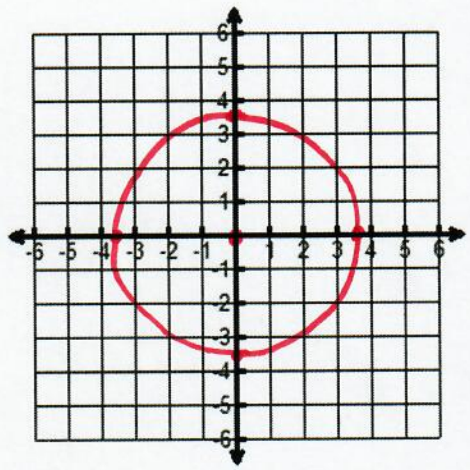
a) $x^2+y^2=36$



Radius: 6

Center: (0,0)

b) $x^2+y^2=13$



Radius: $\sqrt{13} \approx 3.6$

Center: (0,0)

Example: Write the equation of a circle with center at $(0,0)$ and radius 11.

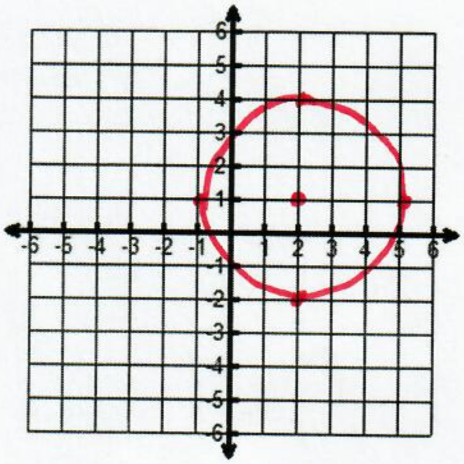
$x^2+y^2=121$

* * *

Equation of a Circle with Center at (h,k) and Radius r: $\boxed{(x-h)^2+(y-k)^2=r^2}$

Examples: Determine the center and radius of each circle, then graph the circle.

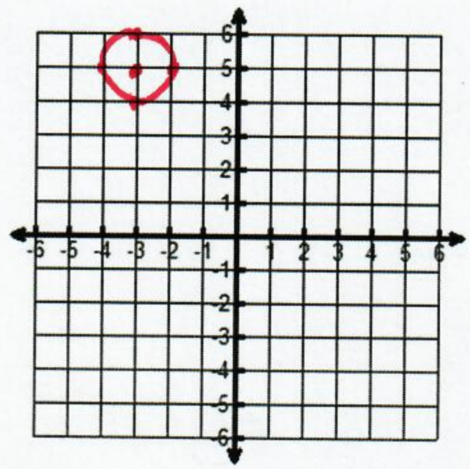
a) $(x-2)^2+(y-1)^2=9$



Radius: 3

Center: (2, 1)

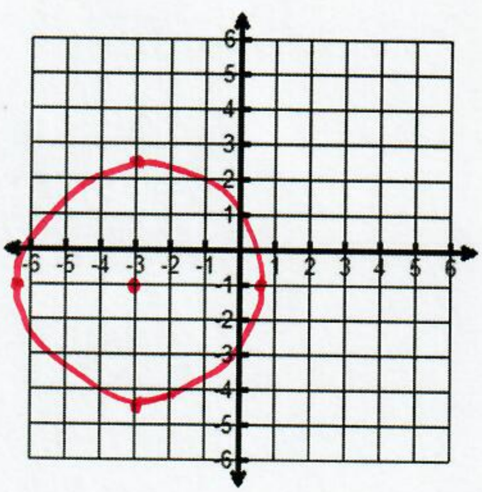
b) $(x+3)^2+(y-5)^2=1$



Radius: 1

Center: (−3, 5)

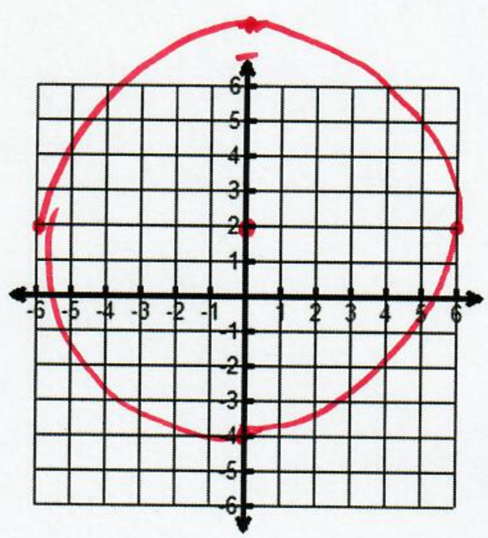
c) $(x+3)^2+(y+1)^2=12$



Radius: $\sqrt{12} \approx 3.46$

Center: (−3, −1)

d) $x^2+(y-2)^2=36$



Radius: 6

Center: (0, 2)

Examples: Write the equation of the circle with the given center and radius.

a) $(2,5)$; $r=7$

Equation: $(x-2)^2+(y-5)^2=49$

b) $(3,-1)$; $r=\sqrt{13}$ $(\sqrt{13})^2=13$

Equation: $(x-3)^2+(y+1)^2=13$

c) $(-2,12)$; $r=15$

Equation: $(x+2)^2+(y-12)^2=225$

d) $(-5,0)$; $r=2\sqrt{3}$ $(2\sqrt{3})^2=12$

Equation: $(x+5)^2+y^2=12$

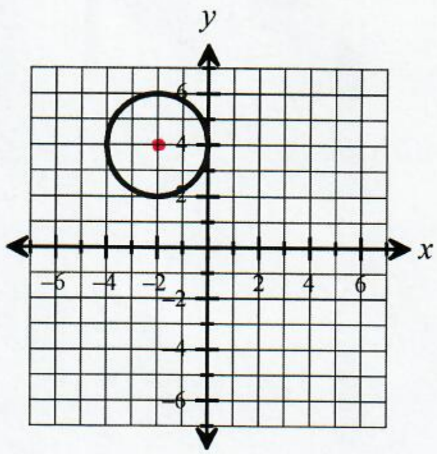
e) $(-6,-9)$; $r=1$

Equation: $(x+6)^2+(y+9)^2=1$

f) $(0,4)$; $r=\frac{1}{2}$

Equation: $x^2+(y-4)^2=\frac{1}{4}$

g)

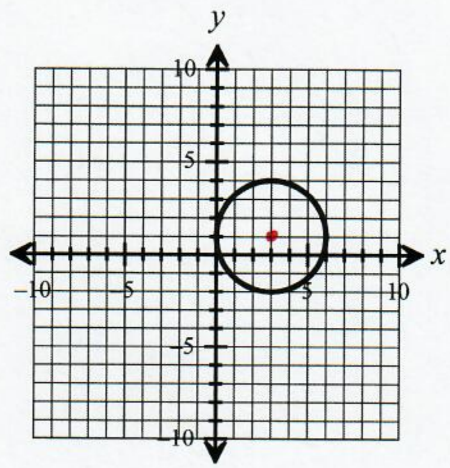


Radius: 2

Center: $(-2,4)$

Equation: $(x+2)^2+(y-4)^2=4$

h)



Radius: 3

Center: $(3,1)$

Equation: $(x-3)^2+(y-1)^2=9$