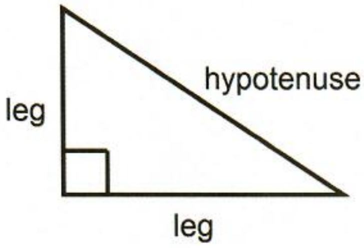
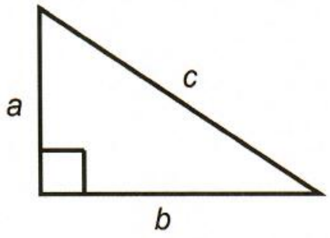
SM 2

Date: **Section: 11.1**

Objective: Pythagorean Theorem

The Pythagorean Theorem: In a **right triangle**, $a^2 + b^2 = c^2$ or $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$.



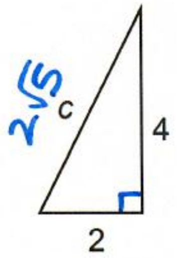


***It does not matter which leg is a or b.

To find the length of the hypotenuse:

Steps for solving the formula for *c*:

1. Substitute *a* and *b* values into the formula.
2. Follow order of operations and do exponents first.
3. Add
4. Square root both sides to get answer.



$2^2 + 4^2 = c^2$

$4 + 16 = c^2$

$\sqrt{20} = \sqrt{c^2}$

$4 \; ⑤$

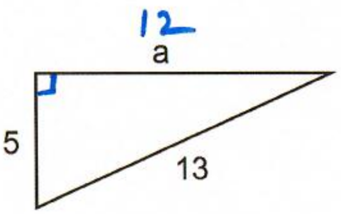
$②②$

$\boxed{2\sqrt{5} = c}$

To find the length of a leg:

Steps for solving the formula for *a or b*:

1. Substitute *a* (or *b*) and *c* into the formula.
2. Square the numbers.
3. Subtract to the other side to get the variable by itself.
4. Square root both sides to get answer.



$a^2 + 5^2 = 13^2$

$a^2 + \cancel{25} = 169$

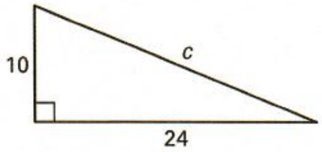
$\quad \cancel{-25} \quad -25$

$\sqrt{a^2} = \sqrt{144}$

$\boxed{a = 12}$

Examples: Find the length of the missing side of each triangle. Write answer as **exact** and **rounded** to the nearest hundredth.

a)



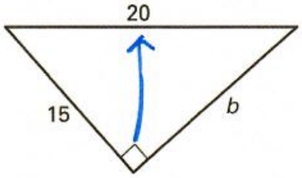
$10^2+24^2=c^2$

$100+576=c^2$

$\sqrt{676}=\sqrt{c^2}$

$\boxed{26=c}$

b)



$15^2+b^2=20^2$

$\cancel{225}+b^2=400$

$-\cancel{225}\qquad -225$

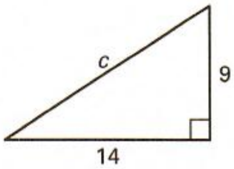
$\sqrt{b^2}=\sqrt{175}$

⑤ 35

⑤ ⑦

$\boxed{\begin{array}{l} b=5\sqrt{7} \\ =13.23 \end{array}}$

c)



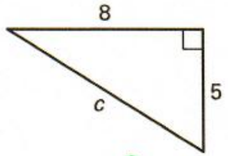
$14^2+9^2=c^2$

$196+81=c^2$

$\sqrt{277}=\sqrt{c^2}$

$\boxed{16.64=c}$

d)



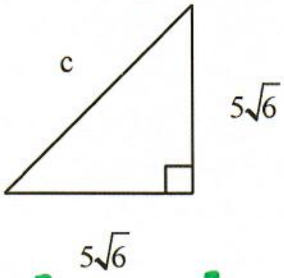
$8^2+5^2=c^2$

$64+25=c^2$

$\sqrt{89}=\sqrt{c^2}$

$\boxed{\begin{array}{l} \sqrt{89}=c \\ 9.43=c \end{array}}$

e)



$(5\sqrt{6})^2+(5\sqrt{6})^2=c^2$

$150+150=c^2$

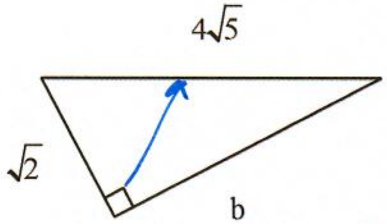
$\sqrt{300}=\sqrt{c^2}$

100 ③

⑩ ⑩

$\boxed{\begin{array}{l} 10\sqrt{3}=c \\ 17.32=c \end{array}}$

f)



$(\sqrt{2})^2+b^2=(4\sqrt{5})^2$

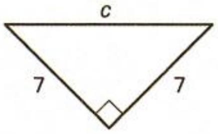
$\cancel{2}+b^2=80$

$-\cancel{2}\qquad -2$

$\sqrt{b^2}=\sqrt{78}$

$\boxed{\begin{array}{l} b=\sqrt{78} \\ =8.83 \end{array}}$

g)



$7^2+7^2=c^2$

$49+49=c^2$

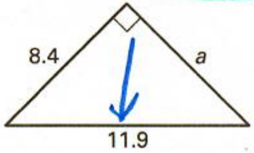
$\sqrt{98}=\sqrt{c^2}$

49 ②

⑦ ⑦

$\boxed{\begin{array}{l} 7\sqrt{2}=c \\ 9.90=c \end{array}}$

h)



$a^2+8.4^2=11.9^2$

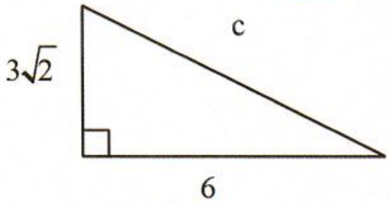
$a^2+70.56=141.61$

$-\cancel{70.56}\qquad -70.56$

$\sqrt{a^2}=\sqrt{71.05}$

$\boxed{a=8.43}$

i)



$(3\sqrt{2})^2+6^2=c^2$

$18+36=c^2$

$\sqrt{54}=\sqrt{c^2}$

9 6

③ ③ ③ ②

$\boxed{\begin{array}{l} 3\sqrt{6}=c \\ 7.35=c \end{array}}$

SM 2

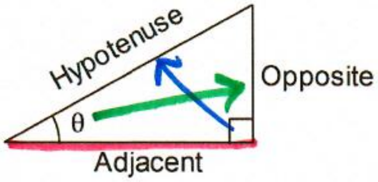
Date: **Section: 11.2**

Objective: Trigonometric Ratios and Inverse Trigonometric Functions

Trigonometry: The study of the relationship between the angles and the sides of a **right triangle**.

Trigonometric Ratio: Special fractions that show how the sides of a right triangle relate to the angles.

3 main or most common trigonometric ratios:



θ = theta

1) **Sine** $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

2) **Cosine** $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

3) **Tangent** $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

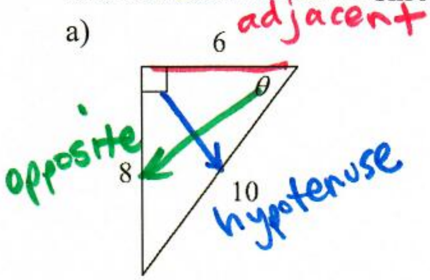
- ★ ***Adjacent*** means " next to θ " (θ is the angle you are focusing on)
- ★ ***Opposite*** means " across from θ " (θ is the angle you are focusing on)
- ★ ***Hypotenuse*** is the side across from the right angle

A common way to remember this is: **SOH-CAH-TOA**

sine opposite hypotenuse cosine adjacent hypotenuse tangent opposite adjacent

Examples: Label the sides as opposite, adjacent, and hypotenuse. Find the lengths of any missing sides. Then find the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

a)

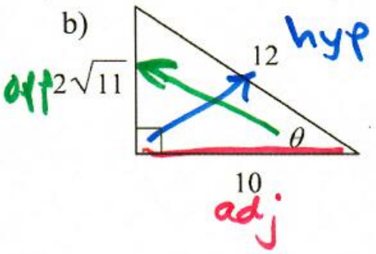


$\sin\theta = \frac{8}{10} = \frac{4}{5}$

$\cos\theta = \frac{6}{10} = \frac{3}{5}$

$\tan\theta = \frac{8}{6} = \frac{4}{3}$

b)

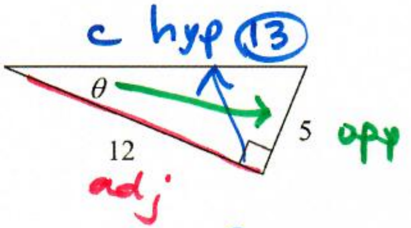


$\sin\theta = \frac{2\sqrt{11}}{12} = \frac{\sqrt{11}}{6}$

$\cos\theta = \frac{10}{12} = \frac{5}{6}$

$\tan\theta = \frac{2\sqrt{11}}{10} = \frac{\sqrt{11}}{5}$

c)



$5^2+12^2=c^2$

$25+144=c^2$

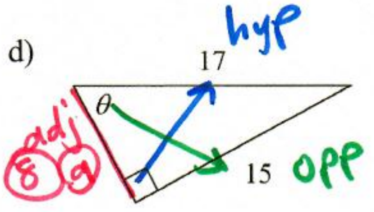
$\sqrt{169}=\sqrt{c^2}$

$13=c$

$\sin\theta = \frac{5}{13}$ $\tan\theta = \frac{5}{12}$

$\cos\theta = \frac{12}{13}$

d)



$a^2 + 15^2 = 17^2$

$a^2 + \cancel{225} = 289$

$\cancel{-225} \quad -225$

$\sqrt{a^2} = \sqrt{64}$

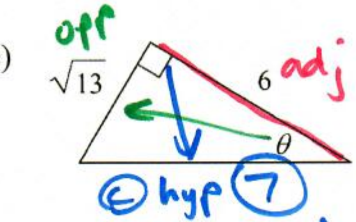
$a = 8$

$\sin\theta = \frac{15}{17}$

$\tan\theta = \frac{15}{8}$

$\cos\theta = \frac{8}{17}$

e)



$(\sqrt{13})^2 + 6^2 = c^2$

$13 + 36 = c^2$

$\sqrt{49} = \sqrt{c^2}$

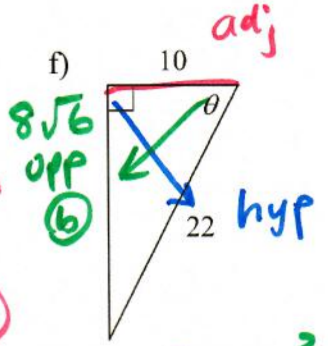
$7 = c$

$\sin\theta = \frac{\sqrt{13}}{7}$

$\cos\theta = \frac{6}{7}$

$\tan\theta = \frac{\sqrt{13}}{6}$

f)



$10^2 + b^2 = 22^2$

$\cancel{-100} + b^2 = 484$

$\quad -100$

$\sqrt{b^2} = \sqrt{384}$

6 64

③②⑧⑧

$b = 8\sqrt{6}$

$\sin\theta = \frac{8\sqrt{6}}{22} = \frac{4\sqrt{6}}{11}$

$\cos\theta = \frac{10}{22} = \frac{5}{11}$

$\tan\theta = \frac{8\sqrt{6}}{10} = \frac{4\sqrt{6}}{5}$

Inverse trigonometric functions:

- **$\sin^{-1}$**
- **$\cos^{-1}$**
- **$\tan^{-1}$**

When do you use inverse functions?

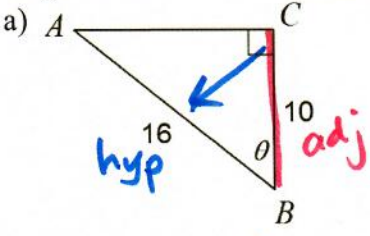
The inverse trig functions allow you to find the **measure of an angle (θ)** in a right triangle.

★ **Make sure your calculator is in DEGREE mode!**

SOH-CAH-TOA

Examples: Label the sides as opposite, adjacent, or hypotenuse. Write an equation involving sine, cosine or tangent. Then find the measure of θ to the nearest tenth of a degree.

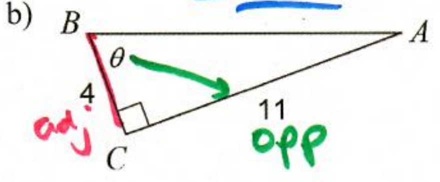
a)



$\cancel{\cos^{-1}\cos}\,\theta = \cos^{-1}\frac{10}{16}$

$\boxed{\theta = 51.3°}$

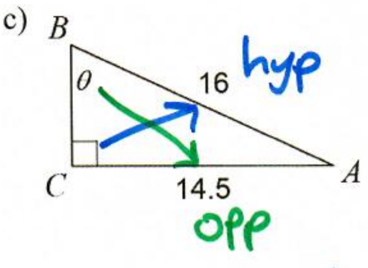
b)



$\cancel{\tan^{-1}\tan}\,\theta = \tan^{-1}\frac{11}{4}$

$\boxed{\theta = 70.0°}$

c)

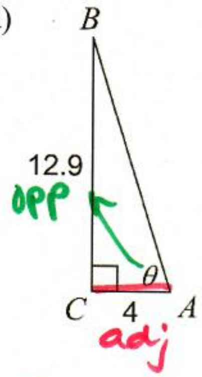


$\cancel{\sin^{-1}\sin}\,\theta = \sin^{-1}\frac{14.5}{16}$

$\boxed{\theta = 65.0°}$

SOH-CAH-TOA

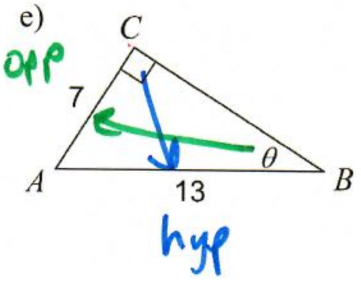
d)



$\tan^{-1}\cancel{\tan}\,\theta = \tan^{-1}\frac{12.9}{4}$

$\boxed{\theta = 72.8°}$

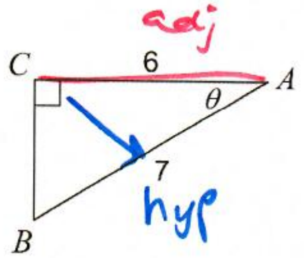
e)



$\sin^{-1}\cancel{\sin}\,\theta = \sin^{-1}\frac{7}{13}$

$\boxed{\theta = 32.6°}$

f)



$\cos^{-1}\cancel{\cos}\,\theta = \cos^{-1}\frac{6}{7}$

$\boxed{\theta = 31.0°}$

SM 2

Date:

Section: 11.3

Objective: Using Trigonometry to find missing side lengths

Steps to find the missing side of a right triangle if you know an angle and one other side:

1) Label the sides is relation to the known angle: opposite, adjacent, and hypotenuse

2) Decide if you need to use sine, cosine, or tangent (use SOH-CAH-TOA).

3) Use SOH-CAH-TOA to write equation.

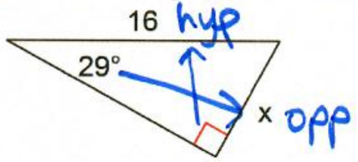
4) Multiply both sides of the equation by the denominator to solve for the variable.

5) Type the problem into the calculator and round to get the missing side.

SOH CAH TOA

Examples: Write an equation involving sine, cosine, or tangent that can be used to find the missing length. Then solve the equation. Round your answers to the nearest tenth.

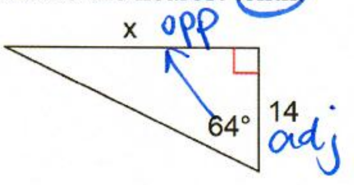
a)



$16 \cdot \sin 29^\circ = \frac{x}{\cancel{16}} \cdot \cancel{16}$

$\boxed{x = 7.8}$

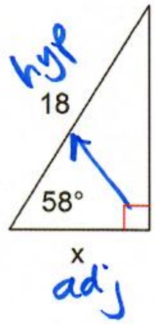
b)



$14 \cdot \tan 64^\circ = \frac{x}{\cancel{14}} \cdot \cancel{14}$

$\boxed{x = 28.7}$

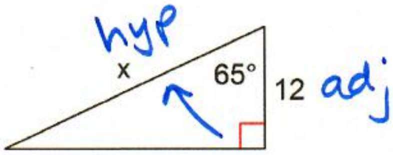
c)



$18 \cdot \cos 58^\circ = \frac{x}{\cancel{18}} \cdot \cancel{18}$

$\boxed{x = 9.5}$

d)

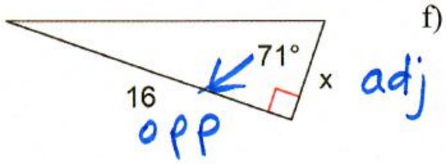


$$X \cdot \cos 65^\circ = \frac{12}{\cancel{x}} \cdot \cancel{x}$$

$$\frac{x\,\cancel{\cos 65^\circ}}{\cancel{\cos 65^\circ}} = \frac{12}{\cos 65^\circ}$$

$$\boxed{x = 28.4}$$

e)

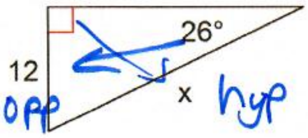


$$x \cdot \tan 71^\circ = \frac{16}{\cancel{x}} \cdot \cancel{x}$$

$$\frac{x\,\cancel{\tan 71^\circ}}{\cancel{\tan 71^\circ}} = \frac{16}{\tan 71^\circ}$$

$$\boxed{x = 5.5}$$

f)

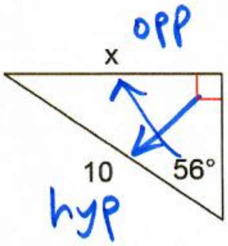


$$x \cdot \sin 26^\circ = \frac{12}{\cancel{x}} \cdot \cancel{x}$$

$$\frac{x\,\cancel{\sin 26^\circ}}{\cancel{\sin 26^\circ}} = \frac{12}{\sin 26^\circ}$$

$$\boxed{x = 27.4}$$

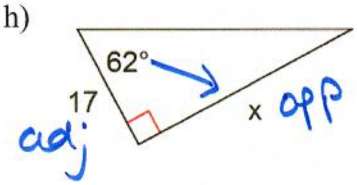
g)



$$10 \cdot \sin 56^\circ = \frac{x}{\cancel{10}} \cdot \cancel{10}$$

$$\boxed{x = 8.3}$$

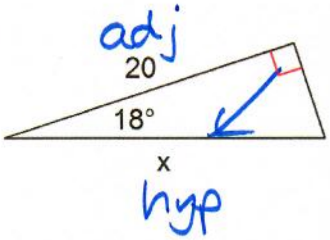
h)



$$17 \cdot \tan 62^\circ = \frac{x}{\cancel{17}} \cdot \cancel{17}$$

$$\boxed{x = 32.0}$$

i)

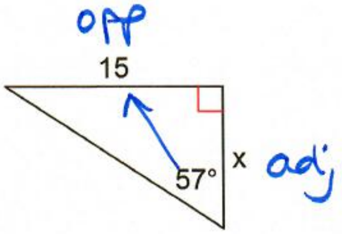


$$x \cdot \cos 18^\circ = \frac{20}{\cancel{x}} \cdot \cancel{x}$$

$$\frac{x\,\cancel{\cos 18^\circ}}{\cancel{\cos 18^\circ}} = \frac{20}{\cos 18^\circ}$$

$$\boxed{x = 21.0}$$

j)

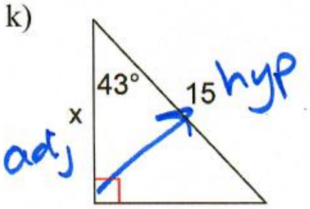


$$x \cdot \tan 57^\circ = \frac{15}{\cancel{x}} \cdot \cancel{x}$$

$$\frac{x\,\cancel{\tan 57^\circ}}{\cancel{\tan 57^\circ}} = \frac{15}{\tan 57^\circ}$$

$$\boxed{x = 9.7}$$

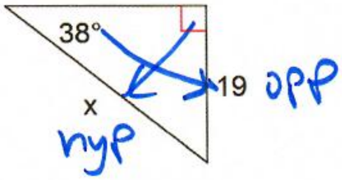
k)



$$15 \cdot \cos 43^\circ = \frac{x}{\cancel{15}} \cdot \cancel{15}$$

$$\boxed{x = 11.0}$$

l)



$$x \cdot \sin 38^\circ = \frac{19}{\cancel{x}} \cdot \cancel{x}$$

$$\frac{x\,\cancel{\sin 38^\circ}}{\cancel{\sin 38^\circ}} = \frac{19}{\sin 38^\circ}$$

$$\boxed{x = 30.9}$$

SM 2

Date:

Section: 11.4

Objective: Solving Story Problems Using Trigonometry

Using trigonometric ratios to solve real world situations.

Vocabulary

Angle of elevation: the angle of the horizontal upward to an object

Angle of depression: the angle from the horizontal downward to an object

* angles of elevation & depression are congruent.

Steps

1) Read the problem, draw a picture, and define the variable.

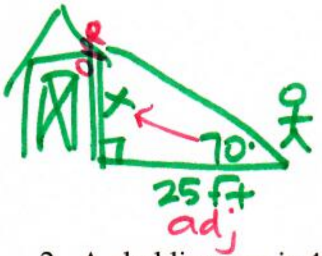
2) Set up the equation based on the given information. Remember SOH-CAH-TOA and inverse trig functions.

3) Solve the equation and SHOW ALL WORK!

4) Label answer, including using correct units.

Read and solve the following.

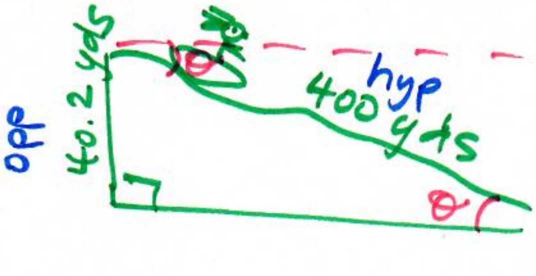
1. A person is 25 feet from the base of a barn. The angle of elevation from the level ground to the top of the barn is 70°. How tall is the barn?



$$25 \cdot \tan 70° = \frac{x}{\cancel{25}} \cdot \cancel{25}$$

$$\boxed{x = 68.69 \text{ feet}}$$

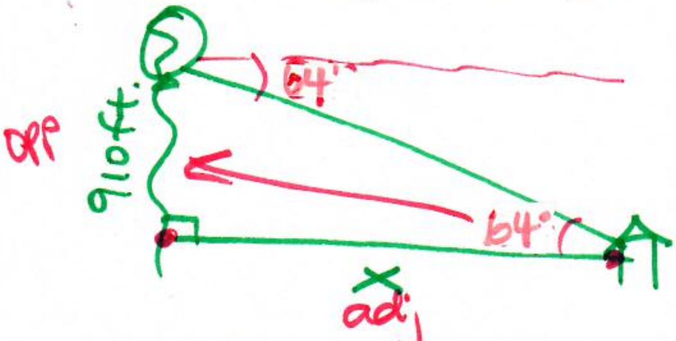
2. A sledding run is 400 yards long with a vertical drop of 40.2 yards. Find the angle of depression of the run.



$$\cancel{\sin^{-1} \sin}\, \theta = \sin^{-1} \frac{40.2}{400}$$

$$\boxed{\theta = 5.77°}$$

3. From a balloon 910 feet high, the angle of depression to the ranger headquarters is 64°. How far is the headquarters from a point on the ground directly below the balloon?

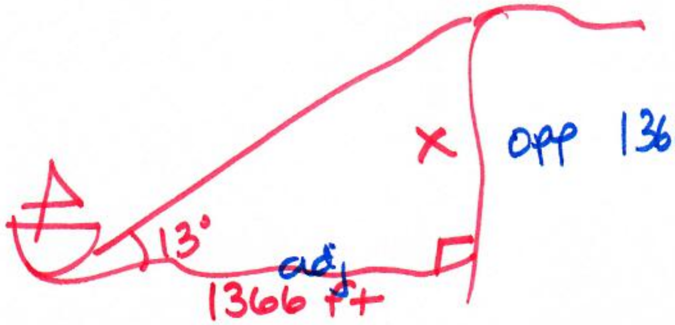


$x \cdot \tan 64° = \frac{910}{\cancel{x}} \cdot \cancel{x}$

$\frac{x \cancel{\tan 64°}}{\cancel{\tan 64°}} = \frac{910}{\tan 64°}$

$\boxed{x = 443.84 \text{ ft}}$

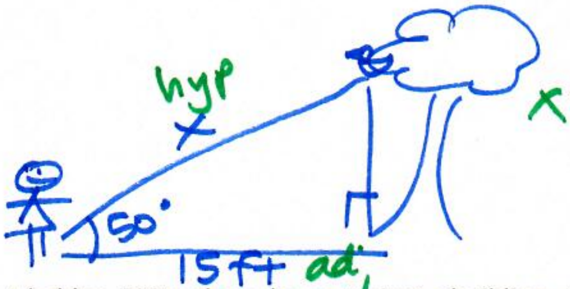
4. From a boat on the lake, the angle of elevation to the top of a cliff is 13°. If the base of the cliff is 1366 feet from the boat, how high is the cliff?



$1366 \tan 13° = \frac{x}{\cancel{1366}} \cdot \cancel{1366}$

$\boxed{x = 315.37 \text{ ft}}$

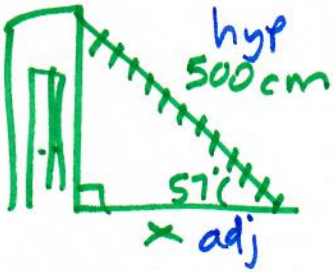
5. A photographer wishes to take a picture of a bird in a tree. She is 15 feet from the base of the tree and is shooting the picture at a $50°$ angle of elevation. How far is the camera from the bird?



$x \cdot \frac{\cancel{\cos 50°}}{\cancel{\cos 50°}} = \frac{\frac{15}{\cancel{x}} \cdot \cancel{x}}{\cos 50°}$

$\boxed{x = 23.34 \text{ ft}}$

6. A ladder, 500cm long, leans against a building. If the angle between the ground and the ladder is 57 degrees, how far from the wall is the bottom of the ladder?



$500 \cdot \cos 57° = \frac{x}{\cancel{500}} \cdot \cancel{500}$

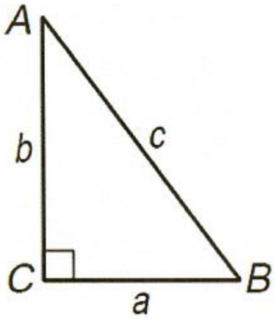
$\boxed{x = 272.32 \text{ cm}}$

SM 2

Date: **Section: 11.5**

Objective: Solving Right Triangles

Solving a Triangle: Figuring out the lengths of all three sides and the measures of all three angles of a triangle. DRAW AND LABEL A TRIANGLE (Label the pieces you know with numbers and the pieces you don't know with variables)! Decide which of these choices your triangle is like. Then follow the instructions. Show work!



- If you know the measure of one angle and the length of one side,
  - To find the measure of the other angle:

    Subtract the known acute angle from 90°

  - To find the lengths of the other sides:

    Use SOH-CAH-TOA to find the missing sides

- If you know the lengths of two of the sides,
  - To find the length of the other side:

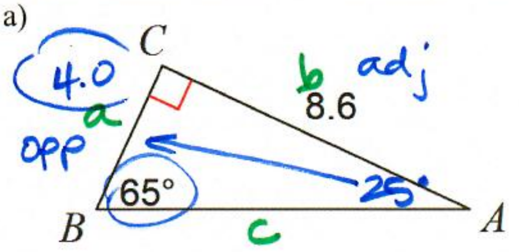
    Use the Pythagorean Theorem

  - To find the measures of the angles:

    Use SOH-CAH-TOA and inverse trig functions

Examples: Solve $\triangle ABC$. Round answers to the nearest tenth. Show all your work.

a)



$m\angle A =$ 25° $a =$ 4.0

$m\angle B =$ 65° $b =$ 8.6

$m\angle C =$ 90° $c =$ 9.5

$8.6\tan 25° = \frac{a}{\cancel{8.6}}\cdot\cancel{8.6}$

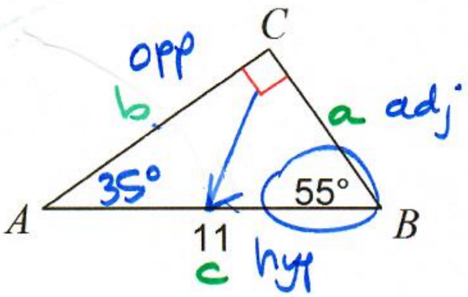
$\boxed{4.0 = a}$

$4.0^2 + 8.6^2 = c^2$

$\sqrt{90.04} = \sqrt{c^2}$

$\boxed{c = 9.5}$

b)



$m\angle A =$ 35° $a =$ 6.3

$m\angle B =$ 55° $b =$ 9.0

$m\angle C =$ 90° $c =$ 11

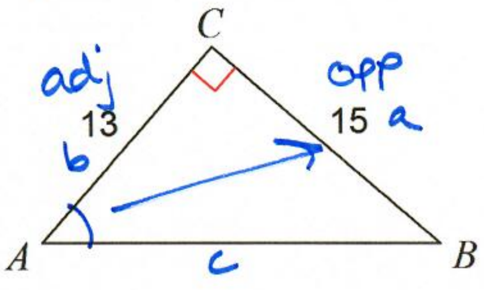
$11\cdot\cos 55° = \frac{a}{\cancel{11}}\cdot\cancel{11}$

$\boxed{6.3 = a}$

$11\cdot\sin 55° = \frac{b}{\cancel{11}}\cdot\cancel{11}$

$\boxed{9.0 = b}$

c)



$m\angle A =$ 49.1° $a =$ 15

$m\angle B =$ 40.9° $b =$ 13

$m\angle C =$ 90° $c =$ 19.8

$15^2 + 13^2 = c^2$

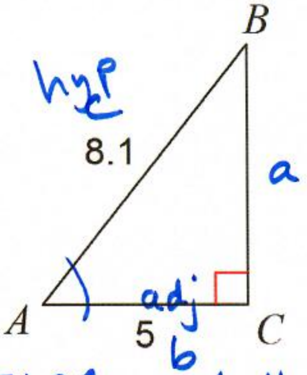
$\sqrt{394} = \sqrt{c^2}$

$\boxed{19.8 = c}$

$\cancel{\tan^{-1}}\cancel{\tan} A = \overset{\tan^{-1}}{\frac{15}{13}}$

$\boxed{A = 49.1°}$

d)



$m\angle A =$ 51.9° $a =$ 6.4

$m\angle B =$ 38.1° $b =$ 5

$m\angle C =$ 90° $c =$ 8.1

$a^2 + 5^2 = 8.1^2$

$a^2 + 25 = 65.61$

$\cancel{-25}\quad -25$

$\sqrt{a^2} = \sqrt{40.61}$

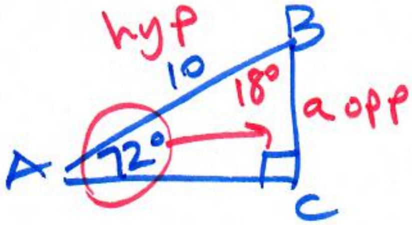
$\boxed{a = 6.4}$

$\cancel{\cos^{-1}}\cancel{\cos} A = \overset{\cos^{-1}}{\frac{5}{8.1}}$

$A = 51.9°$

Draw and label the triangle with the given measurements. Then solve the triangle.

a) $m\angle A = 72°,\ c = 10$



$m\angle A =$ 72° $a =$ 9.5

$m\angle B =$ 18° $b =$ 3.1

$m\angle C =$ 90° $c =$ 10

$$10 \cdot \sin 72° = \frac{a}{\cancel{10}} \cdot \cancel{10}$$

$$\boxed{9.5 = a}$$

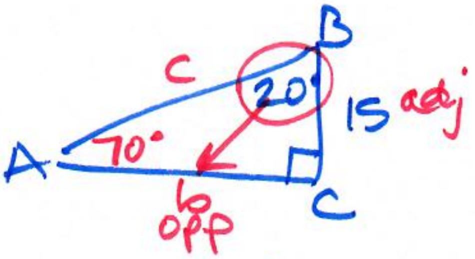
$$9.5^2 + b^2 = 10^2$$

$$\cancel{90.45} + b^2 = 100$$

$$\cancel{-90.45} \qquad -90.45$$

$$\sqrt{b^2} = \sqrt{9.55} \qquad \boxed{b = 3.1}$$

b) $m\angle B = 20°,\ a = 15$



$m\angle A =$ 70° $a =$ 15

$m\angle B =$ 20° $b =$ 5.5

$m\angle C =$ 90° $c =$ $\boxed{16.0}$

$$15 \cdot \tan 20° = \frac{b}{\cancel{15}} \cdot \cancel{15}$$

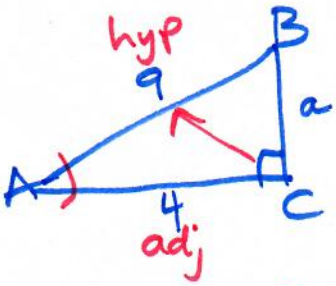
$$b = 5.5$$

$$c \cdot \cos 20° = \frac{15}{\cancel{c}} \cdot \cancel{c}$$

$$\frac{c \cdot \cancel{\cos 20°}}{\cancel{\cos 20}} = \frac{15}{\cos 20}$$

$$\boxed{c = 16.0}$$

c) $b = 4,\ c = 9$



$m\angle A =$ 63.6° $a =$ 8.1

$m\angle B =$ 26.4° $b =$ 4

$m\angle C =$ 90° $c =$ 9

$$a^2 + 4^2 = 9^2$$

$$a^2 + \cancel{16} = 81$$

$$\cancel{-16} \qquad -16$$

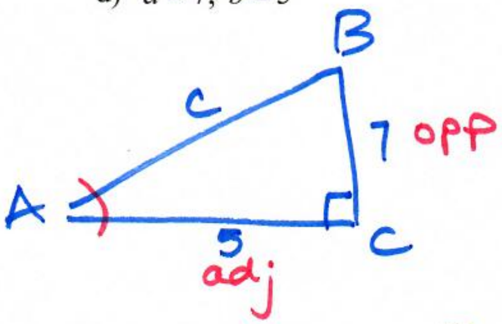
$$\sqrt{a^2} = \sqrt{65}$$

$$\boxed{a = 8.1}$$

$$\cos^{-1} \cancel{\cos} A = \overset{\cos^{-1}}{\frac{4}{9}}$$

$$\boxed{A = 63.6°}$$

d) $a = 7,\ b = 5$



$m\angle A =$ 54.5° $a =$ 7

$m\angle B =$ 35.5° $b =$ 5

$m\angle C =$ 90° $c =$ 8.6

$$7^2 + 5^2 = c^2$$

$$49 + 25 = c^2$$

$$\sqrt{74} = \sqrt{c^2}$$

$$\boxed{8.6 = c}$$

$$\cancel{\tan^{-1}} \cancel{\tan} A = \overset{\tan^{-1}}{\frac{7}{5}}$$

$$\boxed{A = 54.5°}$$