



5.4 Ellipses

Period: _____

* vertices are on the major axis

Locate the vertices and foci of the ellipse (centered at the origin), then graph.

1. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$c^2 = a^2 - b^2$

Center: (0, 0)

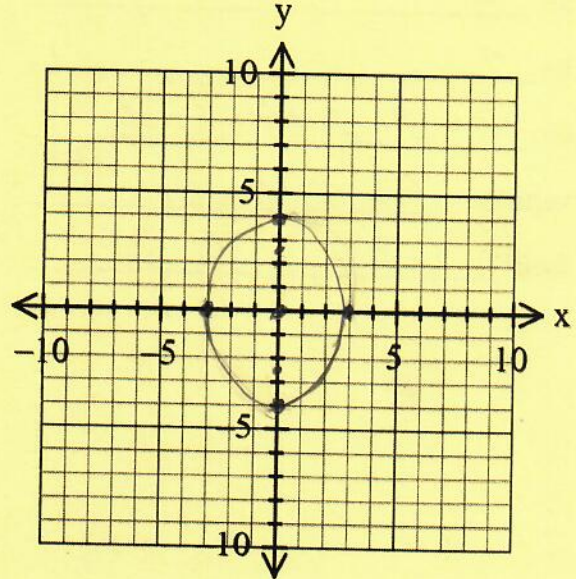
a = 4

b = 3

c = $c^2 = 4^2 - 3^2 = 16 - 9 = \sqrt{7}$ 2.65

vertices: (0, 4); (0, -4)

foci: (0, $\sqrt{7}$); (0, $-\sqrt{7}$)



2. $\frac{x^2}{64} + \frac{y^2}{25} = 1$

Center: (0, 0)

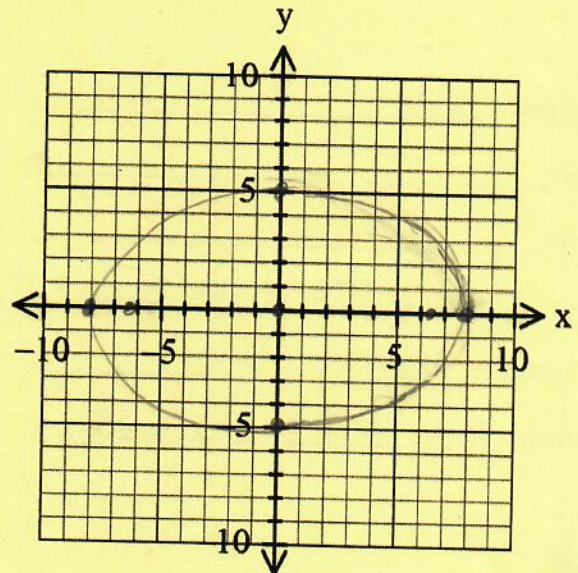
a = 8

b = 5

c = $c^2 = 64 - 25 = \sqrt{39}$

vertices: (-8, 0); (8, 0)

foci: ($-\sqrt{39}$, 0); ($\sqrt{39}$, 0)



$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$3. \frac{4x^2}{36} + \frac{9y^2}{36} = \frac{36}{36}$$

Center: $(0, 0)$

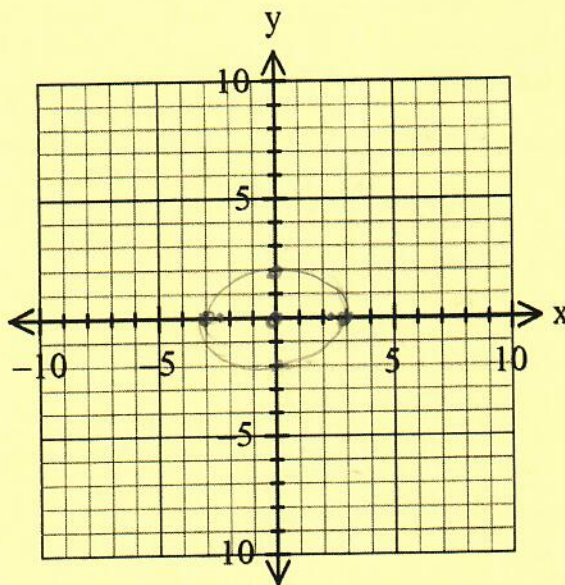
a = 3

b = 2

$$c = c^2 = 3^2 - 2^2 = 9 - 4 = \sqrt{5} \quad 2.24$$

vertices: $(-3, 0); (3, 0)$

foci: $(-\sqrt{5}, 0); (\sqrt{5}, 0)$



$$4. \frac{x^2}{49} + \frac{y^2}{81} = 1$$

Center: $(0, 0)$

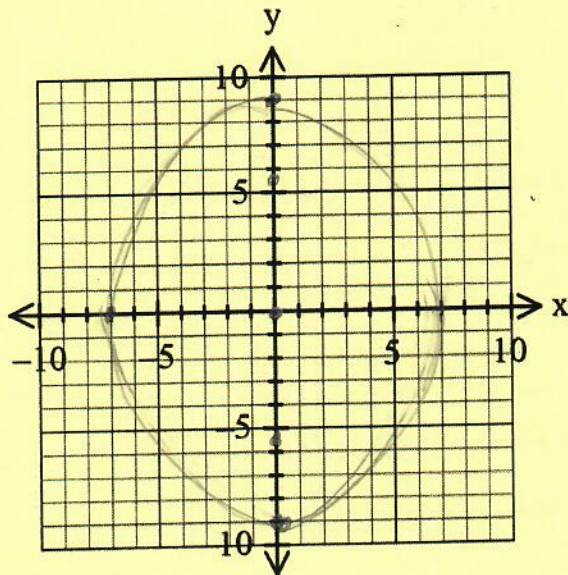
a = 9

b = 7

$$c = c^2 = 9^2 - 7^2 = 81 - 49 = \sqrt{32} = 4\sqrt{2} \quad 5.66$$

vertices: $(0, 9); (0, -9)$

foci: $(0, -4\sqrt{2}); (0, 4\sqrt{2})$



Write an equation in standard form for the ellipse (centered at the origin) that satisfies the given conditions.

5. Foci: $(-6,0)$ and $(6,0)$; Vertices: $(-10,0)$ and $(10,0)$

Which equation should you use?

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Center: $(0,0)$

$a = 10$

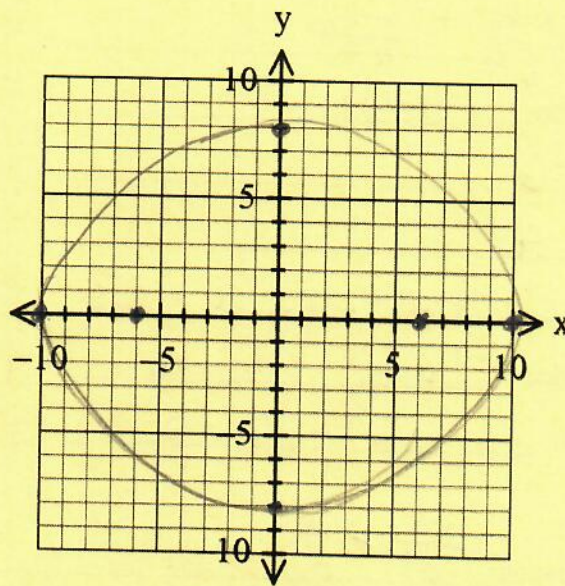
$b = 8$

$c = 6$ $b^2 = 10^2 - 6^2$ $b^2 = 64$ $b = 8$

vertices: $(-10,0)$ $(10,0)$

foci: $(-6,0)$ $(6,0)$

Equation: $\frac{x^2}{100} + \frac{y^2}{64} = 1$



6. Foci: $(0,-3)$ and $(0,3)$; Vertices: $(0,-4)$ and $(0,4)$

Which equation should you use?

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Center: $(0,0)$

$a = 4$

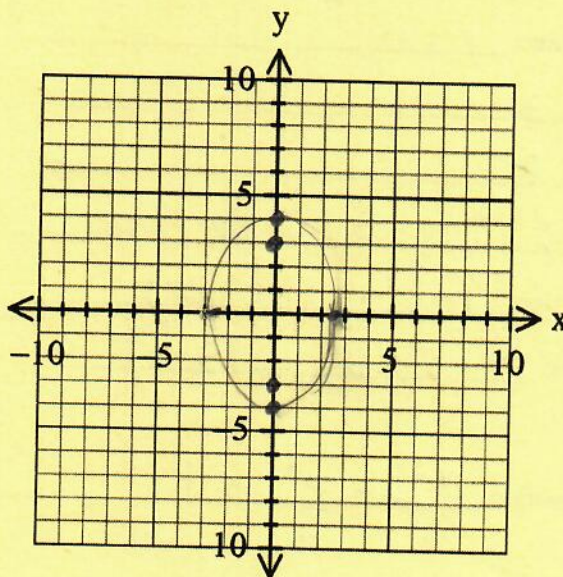
$b = \sqrt{7}$

$c = 3$ $3^2 = 4^2 - b^2$ $b^2 = 7$ $b = \sqrt{7} \approx 2.65$

vertices: $(0,-4)$ $(0,4)$

foci: $(0,-3)$ $(0,3)$

Equation: $\frac{x^2}{7} + \frac{y^2}{16} = 1$



7. Major axis endpoints: (0,6) and (0,-6); Minor axis length: 8 units

Which equation should you use?

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Center: $(0, 0)$

a = 6

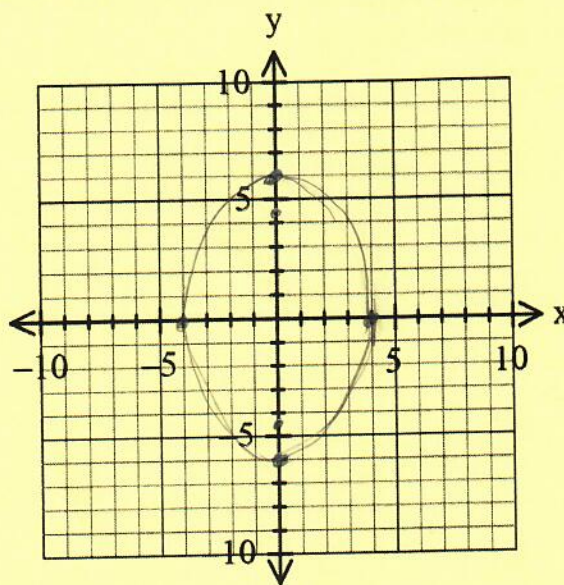
b = 4

$$c = c^2 = 36 - 16 = 20 \quad c = \sqrt{20} = 2\sqrt{5} \quad 4.47$$

vertices: $(0, 6); (0, -6)$

foci: $(0, 2\sqrt{5}); (0, -2\sqrt{5})$

Equation: $\frac{x^2}{16} + \frac{y^2}{36} = 1$



8. Endpoints of axes are: (3,0) & (-3,0) and (0,-2) & (0,2)

Which equation should you use?

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Center: $(0, 0)$

a = 3

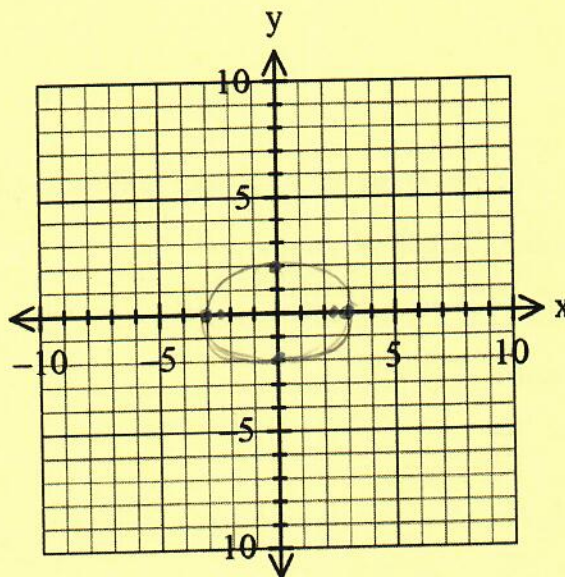
b = 2

$$c = c^2 = 9 - 4 = 5 \quad c = \sqrt{5} \quad 2.24$$

vertices: $(-3, 0); (3, 0)$

foci: $(-\sqrt{5}, 0); (\sqrt{5}, 0)$

Equation: $\frac{x^2}{9} + \frac{y^2}{4} = 1$



Locate the center, vertices and foci of the ellipse, then graph.

9. $\frac{(x-3)^2}{4} + \frac{(y+2)^2}{16} = 1$

Center: $(3, -2)$

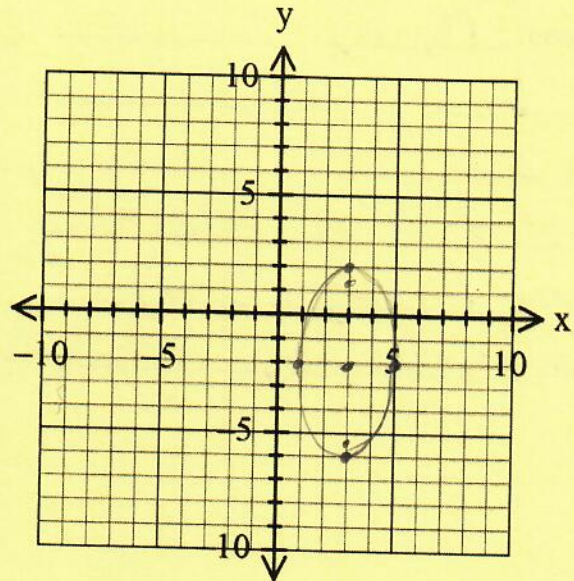
a = 4

b = 2

$c^2 = 16 - 4 = 12$ $c = \sqrt{12} = 2\sqrt{3} \approx 3.46$

vertices: $(3, 2); (3, -6)$

foci: $(3, -2 + 2\sqrt{3}); (3, -2 - 2\sqrt{3})$



10. $\frac{(x+3)^2}{25} + \frac{(y+1)^2}{9} = 1$

Center: $(-3, -1)$

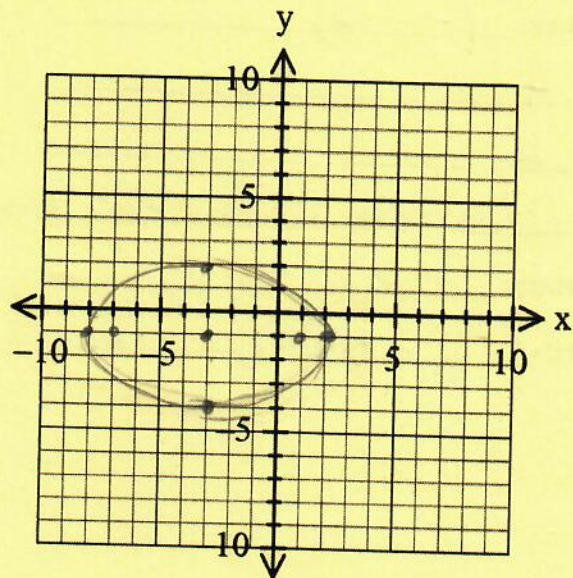
a = 5

b = 3

$c^2 = 25 - 9 = 16$ $c = 4$

vertices: $(-8, -1); (2, -1)$

foci: $(-7, -1); (1, -1)$



$$\frac{(x-3)^2}{36} + \frac{(y+2)^2}{9} = 1$$

$$11. \frac{(x-3)^2}{36} + \frac{(y+2)^2}{9} = \frac{36}{36}$$

Center: $(3, -2)$

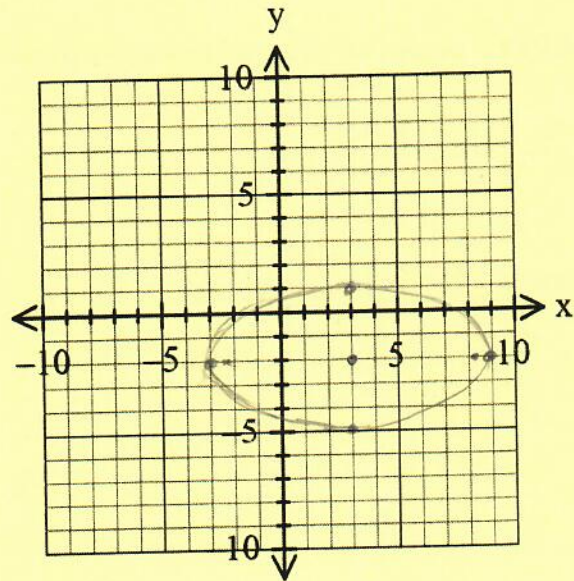
a = 6

b = 3

$$c = c^2 = 6^2 - 3^2 = 27 \quad c = \sqrt{27} = 3\sqrt{3} \quad 5.2$$

vertices: $(-3, -2); (9, -2)$

foci: $(3 - 3\sqrt{3}, -2); (3 + 3\sqrt{3}, -2)$



$$\frac{(x+1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

$$12. \frac{4(x+1)^2}{16} + \frac{(y+2)^2}{16} = \frac{16}{16}$$

Center: $(-1, -2)$

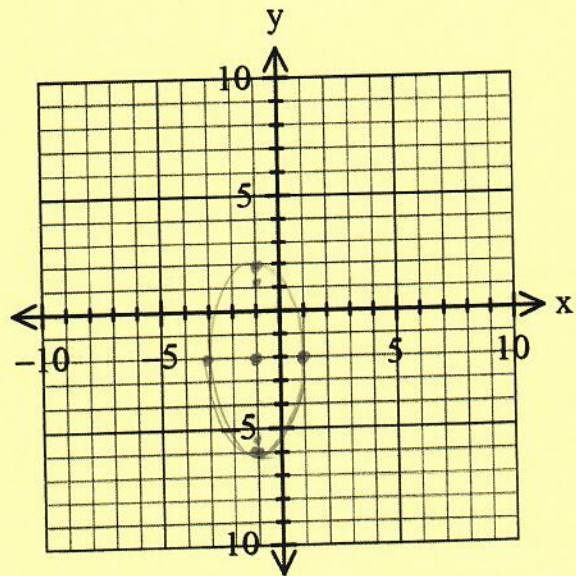
a = 4

b = 2

$$c = c^2 = 16 - 4 = 12 \quad c = \sqrt{12} = 2\sqrt{3} \quad 3.46$$

vertices: $(-1, 2); (-1, -6)$

foci: $(-1, -2 + 2\sqrt{3}); (-1, -2 - 2\sqrt{3})$



Write an equation in standard form for the ellipse that satisfies the given conditions.

13. Foci: (3,-6) and (3,2)
 Vertices: (3,-7) and (3,3)

Which equation should you use?

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Center: $(3, -2)$

a = 5

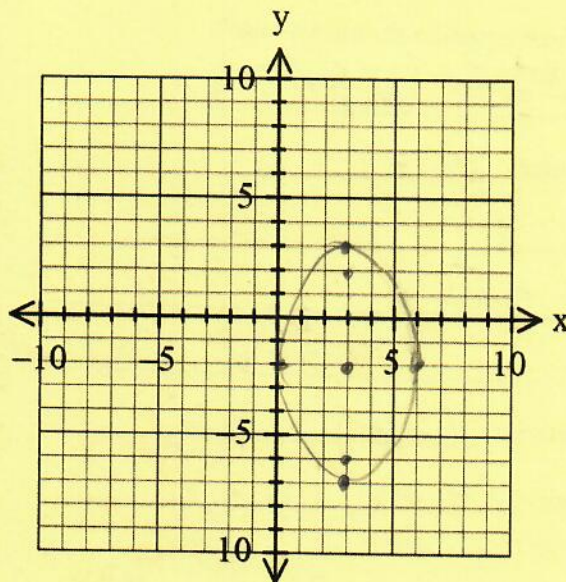
b = 3

c = 4 $16 = 25 - 9$ $b^2 = 9$ $b = 3$

vertices: $(3, -7); (3, 3)$

foci: $(3, -6); (3, 2)$

Equation: $\frac{(x-3)^2}{9} + \frac{(y+2)^2}{25} = 1$



14. Foci: (-5,2) and (3,2)
 Minor axis length is 6.

Which equation should you use?

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Center: $(-1, 2)$

a = 5

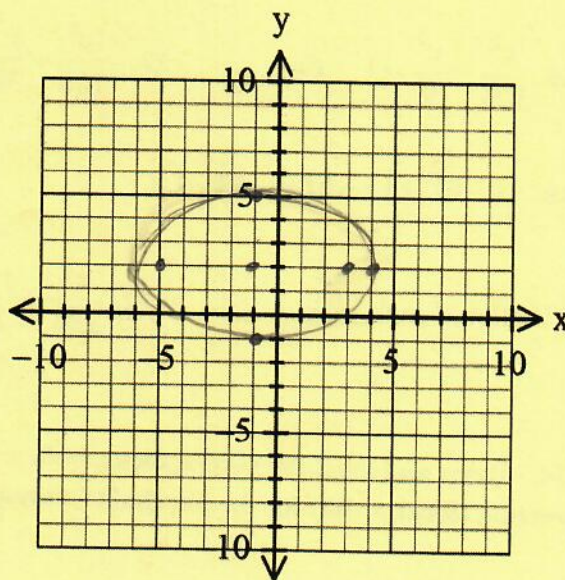
b = 3

c = 4 $16 = a^2 - 9$ $a^2 = 25$ $a = 5$

vertices: $(4, 2); (-6, 2)$

foci: $(-5, 2); (3, 2)$

Equation: $\frac{(x+1)^2}{25} + \frac{(y-2)^2}{9} = 1$



15. Foci: (4,2) and (6,2)
 Vertices: (2,2) and (8,2)

Which equation should you use?

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Center: $(5, 2)$

a = 3

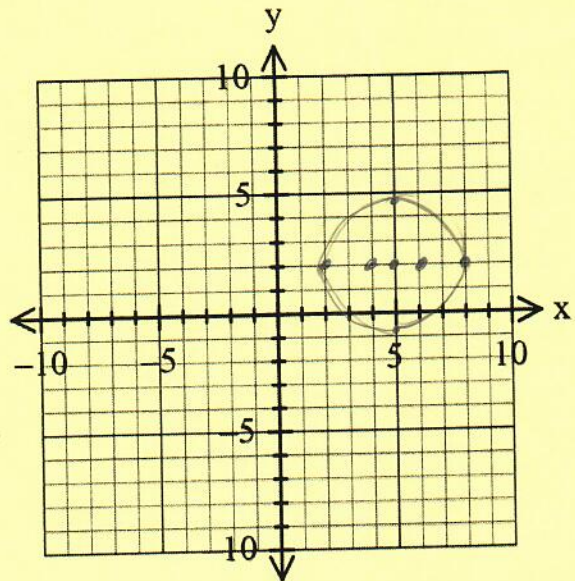
b = $2\sqrt{2}$

c = 1 $c^2 = 3^2 - b^2$ $b^2 = 8$ $b = 2\sqrt{2}$
2.83

vertices: $(2, 2); (8, 2)$

foci: $(4, 2); (6, 2)$

Equation: $\frac{(x-5)^2}{9} + \frac{(y-2)^2}{8} = 1$



REVIEW

Identify each equation as a parabola (p), hyperbola (h), ellipse (e), or circle (c).

16. $\frac{x^2}{121} - \frac{y^2}{9} = 1$ h 17. $\frac{x^2}{100} + \frac{y^2}{36} = 1$ e 18. $y = 8(x-7)^2 + 10$ p

19. $(x-6)^2 + (y-6)^2 = 144$ c 20. $\frac{x^2}{121} - \frac{y^2}{9} = 1$ h

21. $y^2 - x^2 = 4$ h 22. $\frac{x^2}{256} + \frac{y^2}{1} = 1$ e 23. $y = -3x^2 - 4$ p

24. Draw and label the vertex, focus, both a values, axis of symmetry, directrix, and the latus rectum (focal width) for the parabola (estimate where each part is located).

